Profit Sharing and Debt Contracts in Presence of Moral Hazard

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Abstract
This paper compares sharing (equity) and debt contracts in presence of moral hazard which manifests as the hidden effort undertaken by the entrepreneur. The originality of this paper relatively to the existing studies consists in performing the comparison between the two types of contracts while considering a more general context along two dimensions. The first dimension is enabling the internal funds of the entrepreneur to vary between 0% and a level just inferior to 100%. The second dimension is the incorporation of an incentive mechanism to the sharing contract in the context of a two-period relationship. I showed that the sharing and debt contracts are feasible when the internal funds of the entrepreneur are superior to determined thresholds. These thresholds depend on the characteristics of the project (size, payoffs, and probability of success/failure) and the opportunity cost of the financier. **The debt contract is shown to be characterized by larger financial access than the sharing contract**. I have also shown that **the enlargement of the financial-relationship to two periods has an incentivizing effect on the entrepreneur and enlarges the region of financial access for the two types of contracts**, if a common condition of sufficiently foresighted entrepreneur is satisfied. However, two distinct conditions are also necessary for the enlargement of the financial access to occur. For the sharing contract, the second condition is related to the size of the project which should be inferior to a determined threshold. For the debt contract, the second condition is related to the threat of non-renewal of the financing in case of first-period failure, which should be sufficiently stringent. In addition, it has been shown that **the more restrictive the threat of non-renewal the larger the region of financial access**. **However, this is realized at the expense of the second period investment which decreases, and represents the economic efficiency' effect of the debt contract**. Finally, I discussed the effect on the financial access of taxing the "risk-free" financial operation and subsidizing the "higher effort" of the insufficiently-capitalized entrepreneurs.

JEL Codes: D82, D86
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1. Introduction

There is a large literature showing that debt dominates equity contract in presence of information problems and costly monitoring. In Innes (1990) there is asymmetric information between the financier and the entrepreneur about the level of effort undertaken by the latter. Under the conditions of limited liability and the non-decreasing of the financier's payoff, it is shown that debt dominates equity. Once, the second condition is relaxed the optimal contract (which leads to the higher effort) is no more debt but have the form of "live-or-die" having the following feature: the investor is paid a constant share of the firm profit if the latter is below a determined threshold, and nothing when the profit exceeds this level. The explication, of the optimality of debt in case of non-decreasing payoff of the investor is the following: under any other form of contract the entrepreneur will not get the full benefit from the additional payoffs generated out of higher effort, which results in an effort level that does not maximizes the project's payoff. Ul Haque and Mirakhor (1987) reached similar result. The authors consider a two-period model where the loanable funds arise from the saving of a representative consumer facing the classical utility maximization problem in two-period framework with its corollary trade-off between consumption and saving. The investment project is undertaken during the second period. The financier (consumer) is considered as risk averse facing an opportunity cost while the entrepreneur is risk-neutral. The authors show that in the case of certainty debt and equity contracts generate the same level of investment. However, in case of uncertainty on the project's output - which is dependent on the level of effort undertaken by the entrepreneur - the results are different. Indeed, in case of symmetric information between the financier and the entrepreneur (specifically in case of observable level of effort undertaken by the latter) the level of investment is higher in case of profit sharing contract and Pareto sub-optimality does not necessarily occur. However, in presence of moral hazard with unobservable effort the level of investment increases and the return to capital may be lower under sharing contacts.

The other rationale for the dominance of debt over equity is the minimization of monitoring costs as it was shown by the literature on debt contract optimality with costly state verification (e.g. Townsend, 1979 and Gale and Hellwig, 1985). This result is shown when monitoring under equity occurs systematically or what is called deterministic monitoring. Al-Suwailem (2005) develops a one-period model where the effort undertaken by the entrepreneur and the stochastic status of the demand affect the payoff of the project. In this model, the asymmetric information regarding the realized output of the project (which is totally financed by external funds) is revealed by the financier through a random auditing strategy. This strategy reduces the higher monitoring cost of the sharing (equity) contract. Al-Suwailem (2005) shows that equity Pareto-dominates the debt contract for a determined range of the financier's opportunity cost. The range of this Pareto dominance increases with the project's probability of success as well as with the bankruptcy cost. Khan (1987) considers debt and equity contracts in a one-period model and analyzes their Pareto optimality by comparing the expected payoff they generate for the financier and the entrepreneur. In this framework, it is assumed that the investment projects have the same probability distribution but their returns are ex-post uncorrelated. The financier has a continuous utility function and cares only about its expected payoff (which is a less restrictive assumption than the risk neutrality) and the entrepreneurs are risk averse and care about their expected utility. Under symmetric information, it is shown that the equity contract dominates the debt contract because it generates smoother income for the entrepreneur while the financier is indifferent between the two types of contracts for a given sharing rule. In presence of moral hazard, Khan (1987) shows that debt contract dominates the equity contract for sufficiently low level of risk aversion from the side of the financier because it minimizes the cost of monitoring[1]. This trade-off between the benefits of risk-smoothing under equity contracts and the incentive effects of debt contracts has been also shown in Jensen and Mackling (1976), Grossman and Hart (1982) in presence of moral hazard. Trester (1998) develops a four-period model trying to explain why venture capitalists use equity rather than debt to finance entrepreneurial projects. The author considers a framework where information is initially symmetric between the risk-neutral entrepreneur and the venture capitalist. However, subsequently asymmetric information about the payoff of the project arises with a non-null probability which is the case when the entrepreneur learns the status of the project one period before the venture capitalist. In this case, it is shown that debt contract may be infeasible and leads to the use of preferred equity contracts. This is mainly due to the fact that the foreclosure option of the debt contract may incentivize the entrepreneur to behave opportunistically which reduces the expected return of the venture capitalist.

The objective of this paper is to compare equity and debt contracts in presence of moral hazard which manifests as the hidden effort undertaken by the entrepreneur. The originality of this paper relatively to the existing studies consists in performing the comparison between the two types of contracts while considering a more general context along two dimensions. The first dimension is enabling the internal funds provided by the entrepreneur to vary between null (which corresponds to Innes, 1990 and the other above mentioned studies) and a level just inferior to 100%. The justification of considering this more general dimension is the intuition that the opportunistic behavior of the entrepreneur is less likely to happen when his own investment is higher. The second dimension is the incorporation of an incentive mechanism to the equity contract in the context of a two-period relationship. The incentive mechanism is related to the fact that the entrepreneur's (financier') share in the project's payoff is increasing (decreasing) with the internal funds. Therefore, in the context of a two-period relationship the entrepreneur to undertake the higher effort during the first period is even much important since he is not only concerned by the cash-flow generated at the end of the first-period, but also by increasing his share in the second period production cycle. The higher the cash-flow generated in the first period, the higher its share during the second period.

In our model the payoff is observable by the financier without any cost. Besides, the financier and the entrepreneur are risk-neutral. Therefore, an eventual optimality of equity contract relatively to debt will not rest on the existence of random auditing strategy or the smoothing of the uncertain payoffs as it is the case under risk aversion contracting parties. The optimality of equity is rather explored in relation to the reduction of economic inefficiency which emerges under debt contract in the context of two-period relationship.

This economic inefficiency is shown in Dang (2010) who considers two-period debt based relationship between risk-neutral entrepreneur and financier. The entrepreneur has no internal

[1] The model of Khan (1987) is a demand side model which assumes a fixed supply of loanable funds. In this framework, the financier plays a neutral role, and the Pareto Optimality is determined by the preferences of the entrepreneur. Indeed, the author shows that it is always possible to find a sharing rule that makes the financier indifferent between the two types of contracts. This is not realistic since the financier may have an alternative to invest his funds in a secure investment which generate a safe return. In that case, the sharing rule will be constrained by the best alternative opportunity or the opportunity cost of financing the investment projects.

funds and needs a fixed amount of funds to undertake a risky project. The probability of success and failure is dependent on the level of the undertaken effort. Dang (2010) includes two incentivizing tools in the entrepreneur-financier financial contracting. The first incentive is the classical characteristic of the one-period debt contract which consists in making the profits of the entrepreneur non-decreasing in the project's profits[2]. The second incentive for higher effort is the termination threat which consists in the nonrenewal of project financing during the second period in case of failure during the first period.[3] It is clear that this second incentivizing tool penalizes the entrepreneur who undertakes higher effort but whose project failed due to bad luck (realization of the low productivity in the case of high effort). This is clearly a source of economic inefficiency. Dang (2010) argues that this means that an entrepreneur with an investment project having a positive net present value will not be financed during the second period because his first-period project failed.

The two-period debt contract identified in this paper has similar features than that considered in Dang (2010) at the exception that I consider a more general context enabling for the internal funds of the entrepreneur and for the reinvesting of the first-period cash flows. Intuitively, our suggested equity contract will dominate the debt contract in respect to the above mentioned economic efficiency dimension. However, the feasibility of such contract is constrained by the opportunity cost of the lender who might not be able to obtain the expected rate of return otherwise possible under the debt contract. Therefore, another dimension is taken in consideration in our model which is the financial access to external funds by the entrepreneurs according to their available internal resources. I show that the sharing and debt contracts are feasible when the internal funds of the entrepreneur are superior to determined thresholds. These thresholds depend on the characteristics of the project (size, payoffs, and probability of success/failure) and the opportunity cost of the financier. The debt contract is shown to be characterized by larger financial access than the sharing contract. I have also shown that the enlargement of the financial-relationship to two periods has an incentivizing effect on the entrepreneur and enlarges the region of financial access for the two types of contracts, if a common condition of sufficiently foresighted entrepreneur is satisfied. However, two distinct conditions are also necessary for the enlargement of the financial access to occur. For the sharing contract, the second condition is related to the size of the project which should be inferior to a determined threshold. For the debt contract, the second condition is related to the threat of non-renewal of the financing in case of first-period failure, which should be sufficiently stringent. In addition, it has been shown that the more restrictive the threat of non-renewal the larger the region of financial access. However, this is realized at the expense of the second period investment which decreases and represents the economic efficiency' effect of the debt contract. In the policy recommendation section, I discuss the ability of taxing the

[2] Like in Innes (1990) this provides incentives to the entrepreneur to exert high effort to increase the probability of high profits. In the context of the two-state world considered by Dang (2010), increasing the probability of higher profit is not else than decreasing the probability of failure. Therefore, the probability of failure is negatively correlated with the level of undertaken effort.

[3] The role of the nonrenewal of project financing has been also analyzed in the literature about venture capital and monitoring. Sahlman (1990) notes that staged financing is an important tool for venture capitalists to minimize agency costs. Hellmann (1994) shows that, in case of higher uncertainty at the beginning of the project, a venture capitalist provides "staged" finance with the option to terminate a project at interim stages instead of long-term finance. Admati and Pfleiderer (1994) show that the optimal contract in a "venture capitalist"-"entrepreneur" relationship is a fixed-fraction contract whereby the former receives a fixed fraction of the project's payoff and provides financing for that same fraction for future investments.

"risk-free" financial operation and subsidizing the "higher effort" of the insufficiently-capitalized entrepreneurs on the enlargement of the financial access.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 characterizes the profit sharing and debt contracts in the context of a one-period relationship. Section 4 characterizes the two types of contracts in the context of a two-period relationship. Section 5 provides numerical examples that illustrate the theoretical results and explore the differences of two types of financial contracts. In section 6, I discuss some policy implications of the model. An extension of the model is suggested in section 7. Finally, section 8 concludes.

2. The model

I study the contracting relationship between risk-neutral financier and entrepreneur in presence of moral hazard due to the inability of the principal to observe the effort undertaken by the agent after the signature of the financial contract. The entrepreneur is endowed with internal funds but needs complementary external funding to undertake the investment project. In section 3, I study the relationship in the context of one-period whereas a two-period relationship is considered in section 4.

2.1 Economic environment

A risk-neutral entrepreneur operates a firm which generates a stochastic output: a high level $\overline{\pi}$ and a low level $\underline{\pi}$. The probability θ_e of realization of the high output depends on the effort level e undertaken by the entrepreneur. This effort is private information of the entrepreneur and cannot be observed by the risk-neutral financier. However, the latter can observe perfectly the output of the firm. The relationship between the entrepreneur and the financier is analyzed in the context of sharing (equity) and debt contracts.

2.2 Entrepreneur

A risk-neutral entrepreneur is endowed with a technology that produces a stochastic output according to the following distribution

$$\pi = \begin{cases} \overline{\pi} & \text{with a probability of } \theta_e \\ \underline{\pi} & \text{with a probability of } 1-\theta_e \end{cases} \tag{1}$$

where $\overline{\pi}$ and $\underline{\pi}$ represent respectively the high level and low level of output respectively verifying $0 \leq \underline{\pi} < \overline{\pi}$. The probability θ_e is depending on the level of effort $e \in \{h, l\}$ undertaken by the entrepreneur such that the realization of the higher profit is more likely when the high level of effort h takes place: $1 > \theta_h > \theta_l > 0$. The disutility of the effort is captured through the costs c_h and c_l verifying $c_h > c_l \geq 0$ which means that the higher the effort the higher the cost for the entrepreneur. The investment funds needed to operate the firm are represented by F. The entrepreneur is initially endowed with an amount $f \in [0, F[$ which means that he needs complementary external funds of $F - f$ in order to operate the firm. We denote by $x_0 = (F - f)/F$ the share of capital provided by the financier. The remainder share provided by the entrepreneur is therefore $1 - x_0$. We assume that the expected output of the firm is superior to the investment F only in case of higher effort:

$$\begin{aligned} E_\pi^h &= \theta_h \overline{\pi} + (1-\theta_h)\underline{\pi} > F \\ E_\pi^l &= \theta_l \overline{\pi} + (1-\theta_l)\underline{\pi} < F \end{aligned} \tag{2}$$

Since the entrepreneur is assumed to be risk-neutral he increases his utility by maximizing his end of period output after payment of the financier's share and supporting the cost of effort.

Assumption1. *If the entrepreneur could self-finance the firm then he would chose the higher level of effort. This is equivalent to the following condition*

$$E_\pi^h - E_\pi^l = (\theta_h - \theta_l)(\overline{\pi} - \underline{\pi}) > c_h - c_l \tag{3}$$

which signifies that the additional expected revenue resulting from the higher effort exceeds the additional cost.

2.3 Financier

A risk-neutral financier requires an expected rate of return equal to $0 < \rho < 1$ which is the available return on risk-free financial operation. Therefore, the expected payoff of the financier, generated from the investment project, should be equal to $(1+\rho)x_0 F$. While the financier cannot observe the entrepreneur's effort, he observes without cost the firm's output and can infer the entrepreneur's effort choice from the latter's utility maximization problem.

Assumption2. *The expected return of the investment project is higher than the risk-free return if and only if the higher level of effort is undertaken, i.e.:*

$$\frac{E_\pi^l}{F} < 1 + \rho \le \frac{E_\pi^h}{F} \tag{4}$$

3. Financial contracts in a one-period relationship

3.1 Sharing contracts

I consider a one-period relationship which begins at date $t = 0$ and finishes at date $t = 1$. The entrepreneur and the financier agree on a partnership contract $(x_0 F, \alpha, \beta = x_0)$ whereby the entrepreneur commits to undertake the high level of effort $(e = h)$ and invests an amount $f = (1 - x_0)F$ whereas the financier finances the firm by providing an amount $F - f = x_0 F$. The contract stipulates also that the financier receives a share α ($\beta = x_0$) of the output and the entrepreneur receives a share is $1-\alpha$ $(1-\beta)$ in case of success (failure) of the project. We are looking for a sharing contract such that $\alpha \le \beta$ which means that the financier's share in case of success is lower than its share in case of failure of the project[4]. Given this specification, it is clear that in case of failure of the project (realization of the low level of the output $\underline{\pi}$) the financier (entrepreneur) receives a share x_0 $(1-x_0)$ equal to its initial participation to the capital. For the sharing contract to be fully characterized we need to determine the share α .

3.1.1. Symmetric information

In case of symmetric information, the financier observes the effort of the entrepreneur who cannot deviate from its contractual engagement to undertake the high level of effort. In this case, the share α^* that procures the financier an expected rate of return of ρ is given by:

$$\mathrm{E}(\mathrm{W}^{inv}*) = \theta_h \alpha^* \overline{\pi} + (1\text{-}\theta_h)\beta^* \underline{\pi} = (1+\rho)x_0 F \tag{5}$$

Or equivalently

$$\begin{aligned} \alpha^*(x_0) &= x_0[1 - \frac{E_\pi^h - (1+\rho)F}{\theta_h \overline{\pi}}] \\ &= \frac{x_0}{\theta_h \overline{\pi}}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right] \\ &\leq \beta^* = x_0 \end{aligned} \tag{6}$$

This expression shows that if the high expected return E_π^h / F of the project equals the risk-free return $1+\rho$ then $\alpha^* = \beta^* = x_0$. Otherwise, if the high expected return exceeds the risk-free return then the financier accepts a lower share $\alpha^* < \beta^*$ in case of success of the project. It is also clear that the share α^* in case of success (realization of the high output $\overline{\pi}$) increases with the amount of the external financing $x_0 F$. It also increases with the risk-free rate of return ρ . However, the share of the financier decreases when the project become safer. Indeed, it decreases with E_π^h, the higher expected return of the project as well as with θ_h, the probability of success in case of high effort. The entrepreneur utility is given by

$$\mathrm{E}(\mathrm{W}^{ent}*) = \theta_h\left(1-\alpha^*\right)\overline{\pi} + (1\text{-}\theta_h)\left(1-\beta^*\right)\underline{\pi} - c_h \tag{7}$$

[4] From the Islamic Finance perspective, the practice of incentivizing the agent in case of *Mudharabah* (sharing contract with no internal funds from the part of the agent) by increasing his share in the payoffs in case of success of the project, has been approved in the *2nd Islamic Finance Conference* in Kuwait and the fourth *Sharia* opinion of the *1st Al-Barakah Conference.* For the *Musharakah* contract (sharing with financial participation from the principal and the agent) it is known that the sharing of the losses should be proportional to the capital participation of each party but the sharing of the profits could be different according to an initial agreement. Our sharing contract satisfies these criteria. Indeed, in case of failure of the project, the loss $\underline{\pi}-F$ is supported by the financier at the extent of $x_0\underline{\pi}-x_0F$ and by the entrepreneur at the extent of $(1-x_0)\underline{\pi}-(1-x_0)F$. Let's note that if the partnership contract is such that all the investment is provided by the financier ($x_0=1$) than the latter endures the entire loss in case of failure of the project.

3.1.2. Asymmetric information

Assume that the financier offers to the entrepreneur the contract (x_0F, α^*, β) but the entrepreneur deviates from its commitment to undertake the higher effort ($e = h$) and performs the lower effort $(e = l)$. In case of asymmetric information, this deviation is not observable by the financier and could not be inferred from the observation of the output. Indeed, the lower output $\underline{\pi}$ could occur even in the case of higher effort with a probability $1-\theta_h$. Undertaking the lower effort just increases the probability of failure from $1-\theta_h$ to $1-\theta_l$.

Assumption 4. *If the entrepreneur is indifferent between the lower and higher effort then he will fulfills his commitment and will undertake the higher effort $(e = h)$.*

Let's now analyze in which case the deviation of the entrepreneur could occur.

Lemma 1. *The entrepreneur has an incentive to undertake the higher effort $(e = l)$ if his capital participation in the firm $(1-x_0)$ exceeds $(1-\underline{x})$ where $\underline{x}$ is given by*

$$x > \underline{x} = \theta_h \frac{\left(E_\pi^h - E_\pi^l\right) - \left(c_h - c_l\right)}{\left((1+\rho)F - \underline{\pi}\right)(\theta_h - \theta_l)} \tag{8}$$

Proof. See the appendix.

This lemma signifies that the additional cost of effort borne by the entrepreneur ($c_h - c_l$) is lower than his additional expected revenues if he invests at least $(1-\underline{x})F$. Otherwise, (i.e. $1-x_0 < 1-\underline{x}$) the sharing contract (x_0F, α^*, β) is not an incentive-compatible contract since the entrepreneur will be incited to shirk and undertake the lower effort $(e = l)$. Figure 1 shows that the sharing contract is not feasible in case of total external financing of the project $(x_0 = 1)$ which is coherent with the result shown in Innes (1990). However, I show that the sharing contract is feasible when the internal funds of the entrepreneur are sufficiently higher $1 - x > 1 - \underline{x}$.

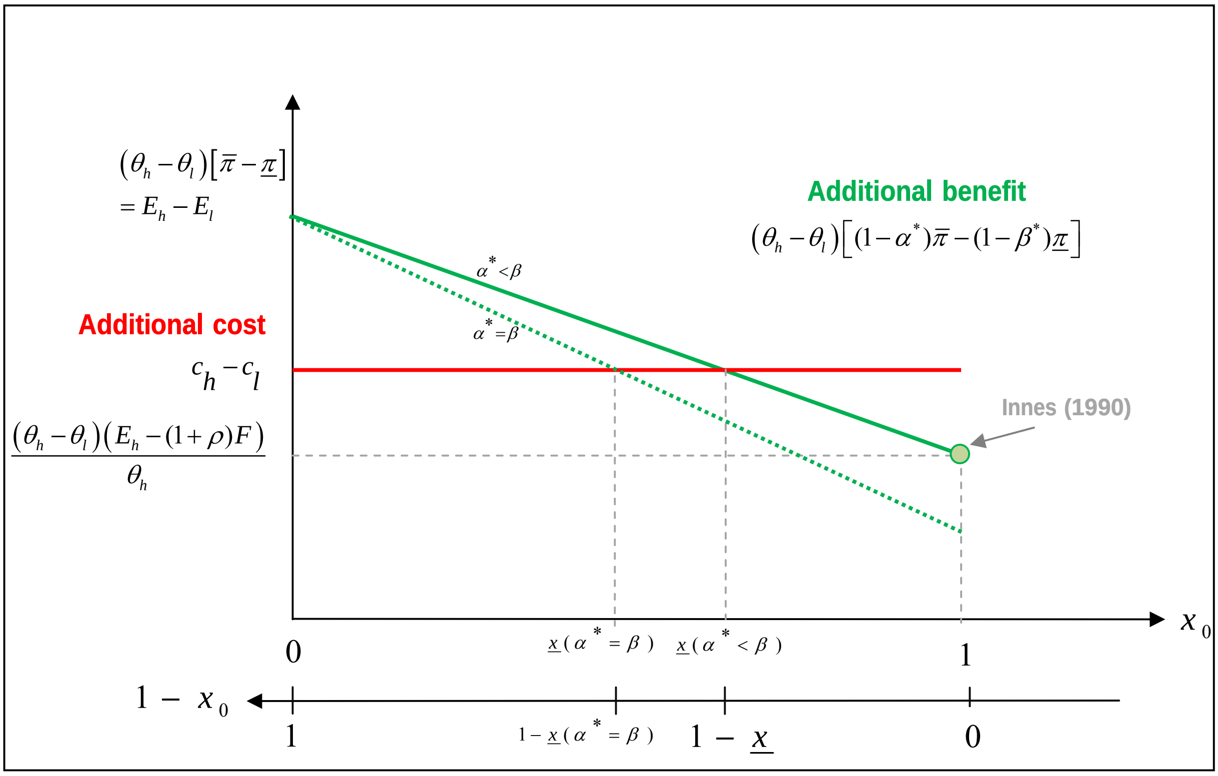

Figure 1. Additional benefit and cost resulting from undertaking higher effort- Case of sharing contract

Figure 1 illustrates also that differentiating the financier's share by fixing a lower share in case of success enlarges the region of feasibility of the sharing contract. The justification of this result rests naturally on the additional incentive to the entrepreneurs endowed with insufficient internal resources to undertake the higher effort.

Proposition 1.

i) *If* $x_0 \le \underline{x}$ *then the equity contract* (x_0F,α^*,β) *provides the financier with an expected rate of return of* ρ.

ii) *If* $x_0 > \underline{x}$ *then*

ii-1) Under the equity contract (x_0F,α^*,β) *the entrepreneur chooses the lower effort and the financier's expected rate of return is* $\tilde{\rho} < \rho$.

ii-2) If $\underline{x} < x_0 \le \max(\underline{x},\hat{x})$ *then the equity contract which provides the financier with an expected rate of return* ρ *is* $(x_0F,\hat{\alpha},\beta)$ *where* $\hat{\alpha}$ *verifies*

$$\alpha^* < \hat{\alpha} = \alpha^*\left(1+\frac{(1+\rho)F}{\theta_l\overline{\pi}}\right) < \beta = x_0 \tag{9}$$

$$\hat{x} = \frac{\theta_l\overline{\pi}}{\theta_l\overline{\pi}+(1+\rho)F} \tag{10}$$

$$\tilde{\rho} = \rho - \left(\frac{\theta_h - \theta_l}{x_0 F}\right)\left(\alpha^* \overline{\pi} - \beta \underline{\pi}\right) \tag{11}$$

Proof. See the appendix.

Figure 2 presents the possible payoffs of the financier under the two equity contracts. The payoff of the entrepreneur could be derived geometrically as the difference between the 45° line and the payoff of the financier.

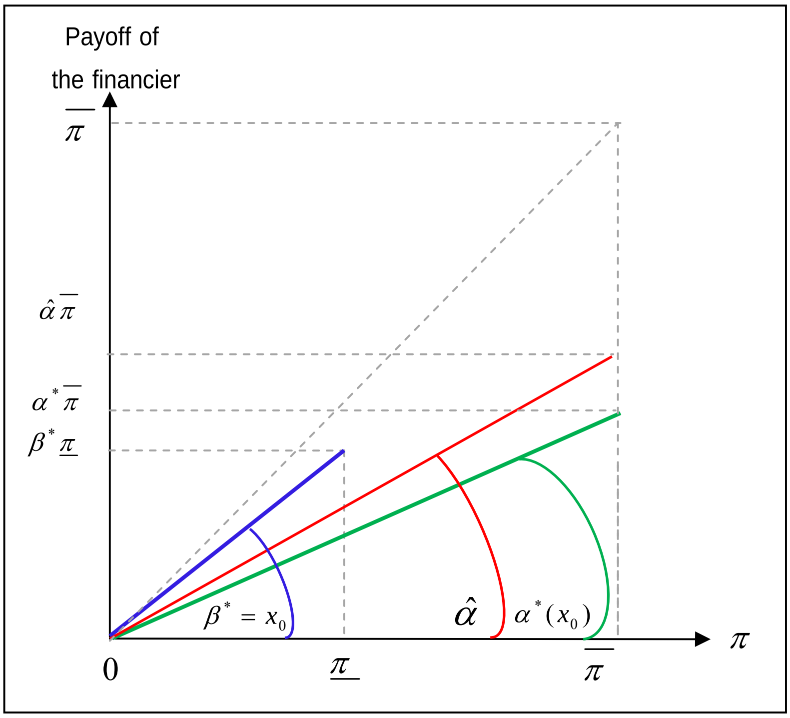

Figure 2. Payoff of the financier according to the three feasible sharing contracts

Figure 3 illustrates the different regions presented in proposition 1 under the condition that the financier requires an expected rate of return of ρ. In the region $[0, \underline{x}]$ the entrepreneur does not require large external financing and has an incentive to undertake the higher effort. In this case, the share of the financier is α^*. However, for $x_0 > \underline{x}$ the amount of entrepreneur's financing is not enough higher and the entrepreneur has an incentive to shirk and undertakes the lower effort.

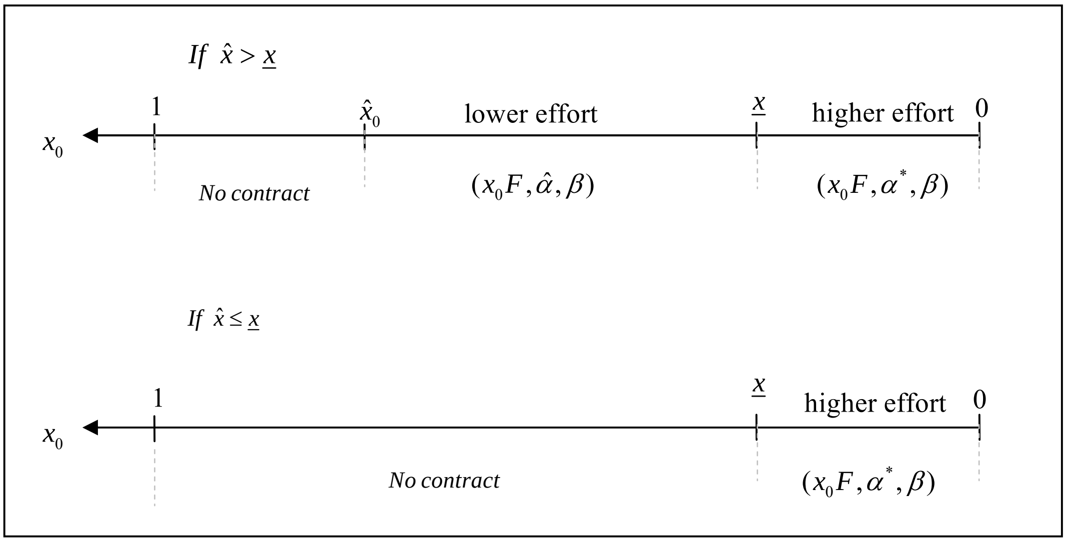

Figure 3. Feasibility and characteristics of the equity contract according to the external financing needs

For this reason, the financier offers in the region $\left]\underline{x}, \max(\underline{x}, \hat{x})\right]$ the contract $(x_0F, \hat{\alpha}, \beta)$ whereby he requires a higher share ($\hat{\alpha} > \alpha^*$) of the payoff in case of success of the project. This result is intuitive since in order for the financier to keep constant its expected return at ρ he has to overcome the decreasing of the probability of success (from θ_h to θ_l) by increasing its share in the outcome from α^* to $\hat{\alpha}$. In the region $\left]\max(\underline{x}, \hat{x}), 1\right]$ the external financing required by the entrepreneur is extremely higher so that no equity contract is feasible due to the inability of the financier to incentivize the entrepreneur to undertake the higher effort, or to increase its share to the level that generates an expected return equal to the risk-free rate of return.

3.2 Debt contract

Let's know consider that the financier signs with the entrepreneur a debt contract $(x_0F, \min(x_0FZ, \pi))$ whereby the financier provides a financing x_0F at a gross rate of interest equal to $Z > 1$. At the end of the period, if the payoff of the firm π is superior to the sum of the principal and interest (x_0FZ) then the financier receives a fixed payment of x_0FZ and the entrepreneur's payoff equals $\pi - x_0FZ$. Otherwise, the financier recuperates the entire output π. It is clear that we have to ensure that $\overline{\pi} > x_0FZ$ in order for the financier to be reimbursed (the principal and interest) in case of success of the project ($\pi = \overline{\pi}$). If $\underline{\pi} \geq x_0FZ$ then the financier receives x_0FZ with certitude even in case of failure of the project. In the case $\underline{\pi} < x_0FZ \leq \overline{\pi}$ the payment received by the financier is given by:

$$W^{inv} = \min(x_0 FZ, \pi) = \begin{cases} x_0 FZ & \text{with a probability of } \theta_e \\ \underline{\pi} & \text{with a probability of } 1 - \theta_e \end{cases} \tag{12}$$

and the cash-flow which remains for the entrepreneur is the following:

$$\pi - \min(x_0 FZ, \pi) = \begin{cases} \overline{\pi} - x_0 FZ & \text{with a probability of } \theta_e \\ 0 & \text{with a probability of } 1 - \theta_e \end{cases} \tag{13}$$

Figure 4 illustrates the "classical" payoff of the financier under the debt contract with specific annotation of the two-state output of the project in our model. It is clear that the financier recuperates all the output in case of failure of the project.

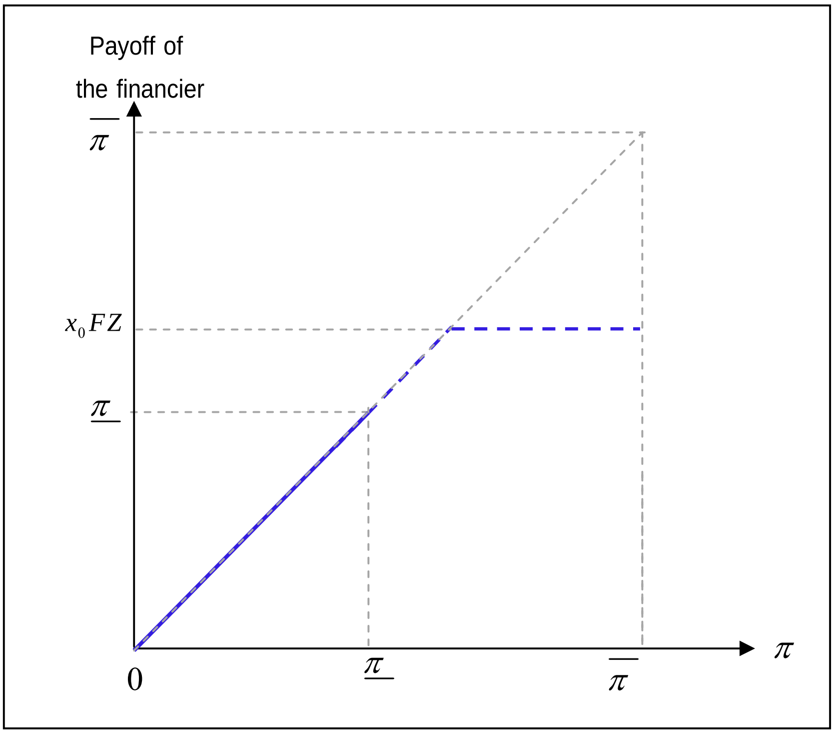

Figure 4. Payoff of the financier under the debt contract

Proposition 2. *The feasibility and characteristics of the debt contract depend on the amount of external financing as follows:*

- *If* $x_0 \in \left[0, \underline{\pi}/(1+\rho)F\right]$ *the contract is* $(x_0F, (1+\rho)x_0F)$.
- *If* $x_0 \in \left]\underline{\pi}/(1+\rho)F, \min(x_h, x_l^{'})\right]$ *the contract is* $(x_0F, Z_h x_0F)$.
- *If* $x_0 \in \left]\min(x_h, x_l^{'}), x_l^{'}\right]$ *the contract is* $(x_0F, Z_l x_0F)$.
- *If* $x_0 \in \left]x_l^{'}, \max(x_h, x_l^{'})\right]$ *the contract is* $(x_0F, Z_h x_0F)$.
- *If* $x_0 \in \left]\max(x_h, x_l^{'}), 1\right]$ *there is no feasible debt contract.*

  with

$$(1+\rho) < Z_h < Z_l \tag{14}$$

  The expressions of x_l, $x_l^{'}$, x_h Z_h and Z_l which depend on the characteristics of the project, the risk-free rate of return and the marginal cost of high effort, are given in the appendix.

Proof. See the appendix.

Figure 5 illustrates the different regions presented in proposition 2.

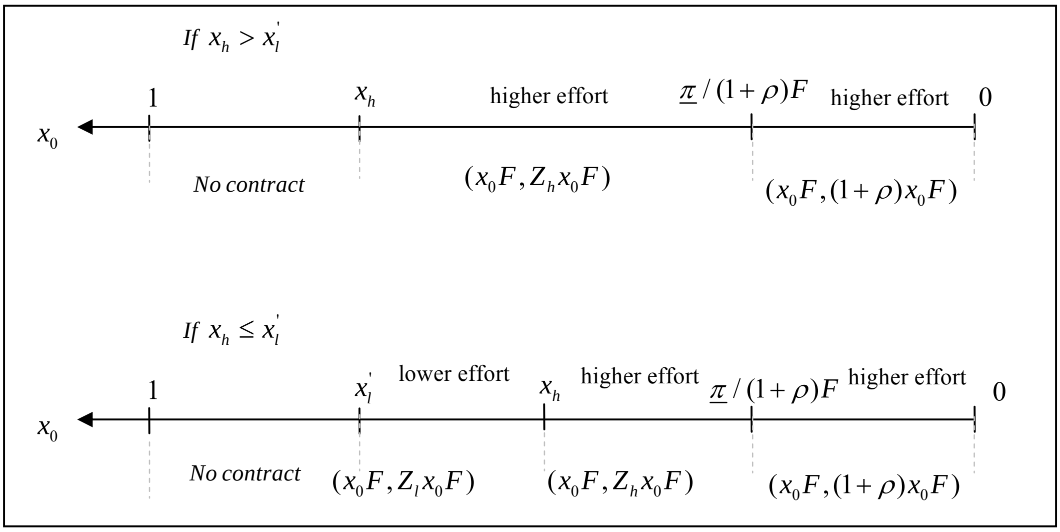

Figure 5. Feasibility and characteristics of the debt contract for different levels of the external financing needs

In the region $\left[0, \underline{\pi}/(1+\rho)F\right]$ the principal and interests of the loan borrowed by the entrepreneur could be reimbursed with certainty whatever the outcome of the project. For this reason the gross interest rate is equal to the risk-free gross rate of return $(1+\rho)$. However, for $x_0 > \underline{\pi}/(1+\rho)F$ the entrepreneur defaults on the loan in case of failure of the project. Therefore, the financier provides the entrepreneur with the required financing in the region

$\left]\underline{\pi}/(1+\rho)F, x_h\right]$ at a higher gross interest rate $Z_h > (1+\rho)$ and it is shown that, in this region the entrepreneur has an incentive to undertake the higher effort. However, when the internal resources are sufficiently low (or equivalently the externally required financing is sufficiently high $x > x_h$) the entrepreneur has an incentive to shirk and undertakes the lower effort. For this reason the gross interest rate in the region $\left]x_h, x_l^{'}\right]$ increases to $Z_l > Z_h > (1+\rho)$ when $x_h \leq x_l^{'}$ and the debt contract is not feasible in the region $\left]\max(x_h, x_l^{'}), 1\right]$.

4. Financial contracts in a two-period relationship

The characteristics of the firm are identical to those described in section 2. I consider an entrepreneur who is initially ($t = 0$) endowed with an amount of capital equal to $f = (1 - x_0)F$. At the beginning of the second period the entrepreneur will reinvest any cash-flow $(1 - x_1)F$ generated from the first-period project in the new project. It is particularly interesting to analyze if the larger horizon of the relationship, relatively to the one-period horizon considered in section 3, will incentivize the entrepreneur to undertake the higher effort and consequently if it enables the enlargement of the region of external financial access. Indeed, if the initial amount $f = (1 - x_0)F$ of capital endowment of the entrepreneur verifies

$$x_0 \in \left]\max(\underline{x}, \hat{x}), 1\right] \tag{15}$$

$$x_0 \in \left]\max(x_h, x_l^{'}), 1\right] \tag{16}$$

then according to the results of section 2 the entrepreneur could not benefit from external funding in the context of a one-period relationship and therefore he could not undertake his project. I will analyze which type of financing contract (sharing or debt) enables the access to finance with lower constraint in regards to the initial funding and which contract will succeed to incentivize the undertaking of higher effort.

4.1 Sharing contract

The partnership between the financier and the entrepreneur covers now two periods. At the initial date $t = 0$ the two parties agree on two separate partnership contracts. At the beginning of the first period ($t = 0$) the entrepreneur and the financier agree on a partnership contract $(x_0 F, \widehat{\alpha}_1, \widehat{\beta}_1 = x_0)$ whereby the entrepreneur commits to undertake the high level of effort $(e = h)$ and participates with an amount of capital of $f = (1 - x_0)F$ whereas the financier finances the firm by providing an amount $F - f = x_0 F$. The entrepreneur commits to reinvest his share of the payoff during the second period so that the second partnership contract becomes $(x_1 F, \widehat{\alpha}_2, \widehat{\beta}_2 = x_1)$ where

$$\left(1-x_1\right)F=\begin{cases}\left(1-\widehat{\alpha}_1\right)\overline{\pi} & \text{with prob } \theta_e\\ \left(1-\widehat{\beta}_1\right)\underline{\pi} & \text{with prob } 1\text{-}\theta_e\end{cases} \tag{17}$$

Condition (17) states that the self-financed capital of the entrepreneur during the second period arises from the payoff he receives at the end of the first period. We have also to ensure that even in the case the entrepreneur is incentivized to undertake the higher effort, the expected wealth of the financier in each period is equal to wealth he would obtain if he invests his capital in a risk-free asset. Therefore, the following conditions should hold:

$$\mathrm{E}(\widehat{\mathrm{W}}_1^{inv})=\theta_h\widehat{\alpha}_1\overline{\pi}+(1-\theta_h)\widehat{\beta}_1\underline{\pi}=(1+\rho)x_0F \tag{18}$$

$$\mathrm{E}(\widehat{\mathrm{W}}_2^{inv})=\theta_h\widehat{\alpha}_2\overline{\pi}+(1\text{-}\theta_h)\widehat{\beta}_2\underline{\pi}=(1+\rho)x_1F \tag{19}$$

Therefore, we obtain the shares that characterize the two partnership contracts are given by

$$\widehat{\alpha}_1=\alpha^*(x_0) \tag{20}$$

$$\widehat{\alpha}_2=\alpha^*(x_1) \tag{21}$$

Where $\alpha^*(x_0)$ is given by (6). Let's note that at date $t=0$ the share $\widehat{\alpha}_2$ that characterizes the second-period partnership contract is not fully determined. Only its variation with the payoff $\left(1-x_1\right)F$ according to (21) and (6) is known. Using (21), (17) and (6) we obtain the ex-ante (at $t=0$) expression of the share $\widehat{\alpha}_2(e_1=e)$ which depends on the state of the nature that takes place at $t=1$ and the effort undertaken by the entrepreneur during the first period:

$$\widehat{\alpha}_2(e_1=e)=\begin{cases}\widehat{\alpha}_2^{up}=\dfrac{F-\left(1-\widehat{\alpha}_1\right)\overline{\pi}}{\theta_h\overline{\pi}F}\left((1+\rho)F-(1-\theta_h)\underline{\pi}\right) & \text{with prob } \theta_e\\ \widehat{\alpha}_2^{down}=\dfrac{F-\left(1-x_0\right)\underline{\pi}}{\theta_h\overline{\pi}F}\left((1+\rho)F-(1-\theta_h)\underline{\pi}\right) & \text{with prob 1-}\theta_e\end{cases} \tag{22}$$

Given that the share $\widehat{\alpha}_2$ is decreasing with the payoff ($\overline{\pi}$ or $\underline{\pi}$) that takes place during the first period, it is possible that the entrepreneur will be incentivized to undertake the higher effort during the first period in order to maximize its second period payoff. Therefore, the decision of the entrepreneur should be analyzed by considering its expected inter-temporal discounted wealth

$$EW^{ent}(e_1,e_2)=-c_{e_1}+\delta EW_2^{ent}(e_1,e_2) \tag{23}$$

where $\delta<1$ is the entrepreneur's discount factor. Equation (23) shows that the effort $e_1\in\{l,h\}$ undertaken by the entrepreneur requires a cost c_{e_1} and affects the expected wealth $EW_2^{ent}(e_1,e_2)$ during the second period since it impacts the probabilities of success and failure of the project and therefore the revenues that will be reinvested during the second period and consequently the probability that his share in the second's period project equals $\widehat{\alpha}_2^{up}$ or $\widehat{\alpha}_2^{down}$.

Proposition3. *The partnership over two periods enlarges the region of financial access to* $x_0 \in \left[\max(\underline{x}, \hat{x}), \tilde{x}\right]$ *and incentivizes the entrepreneur to undertake the higher effort* $(e_1 = h, e_2 = h)$ *if the latter is sufficiently foresighted and the size of the project inferior to a determined threshold:*

$$\delta > \begin{cases} \tilde{\delta} & \text{If } F < \underline{F} \\ \dfrac{\underline{\underline{x}} - \underline{x}}{\underline{\underline{x}} - \hat{x}}\tilde{\delta} & \text{If } \underline{F} \le F \le \max(\underline{F}, \underline{\underline{F}}) \end{cases}$$

where the thresholds $\tilde{\delta}, \tilde{x}, \underline{\underline{x}}, \underline{F}$ and $\underline{\underline{F}}$ *are defined in the appendix.*

Proof. See the appendix.

This result is intuitive signaling that if the entrepreneur does not put sufficient importance on the payoffs that he will obtain during the second period, then no additional incentive will result from the larger horizon of the relationship financier/entrepreneur. Meanwhile, even if the entrepreneur is sufficiently foresighted but the project's size exceeds $\max(\underline{F}, \underline{\underline{F}})$ then the financier will not provide financing for the projects that are not financially feasible over a one-period relationship.

4.2. Debt Contract

The debt contract is characterized by an incentive mechanism which has been extensively analyzed in the financial contracting literature (see for example, Innes, 1990). Under this mechanism the payoff of the entrepreneur after repayment of the debt is increasing in the firm output. Indeed, in case of success of the project, the entrepreneur gets the full benefit from the additional payoffs generated out of higher effort. As a consequence, the entrepreneur has an incentive to maximize his expected output by undertaking the higher effort. However, I showed in Proposition 2 that there is no feasible debt contract in the region$]\max(x_h, x_l^{'}),1]$ signifying that this incentive mechanism is inefficient in preventing the entrepreneur from undertaking the low level of effort when his contribution is too low. This is particularly the case when the marginal cost of higher effort exceeds a determined threshold. In this section I consider the relationship between the financier and the entrepreneur in the context of two periods. I endow the debt contract with a second incentive mechanism which has been suggested by Dang (2010)[5] and consists in incorporating the threat of non-renewal of the financing during the second period in case of first-period's failure of the project. At the initial date $t = 0$, the two parties agree on the debt contract $(x_0 F, \min(x_0 F Z_1, \pi))$ which covers the

[5] This has been suggested in a different two-period horizon where the entrepreneur has no initial endowment and is not enabled to reinvest his first-period residual cash-flow.

first period. In case of success of the project at the end of the first period, the financier renews financing the project during the second period with certainty. Otherwise (i.e. in case of failure of the project) the financier renews the financing with probability $0 < \underline{p} < 1$. Therefore, the second debt contract is $(\underline{p}, x_1 F, \min(x_1 F Z_2, \pi))$ where $(1 - x_1)F$ represents the self-financing of the entrepreneur financed with the first-period payoff and given by:

$$(1 - x_1)F = \overline{\pi} - x_0 F Z_1 \tag{24}$$

The objective is to analyze in which case the new form of relationship incentivizes the entrepreneur to undertake the high level of effort during the first period and enlarges the region of financial access.

Proposition4.

The debt contract relationship over two periods enlarges the region of financial access to $x_0 \in \left[\max(x_h, x_l^{'}), \overline{\overline{x}}(\delta)\right]$ *and incentivizes the entrepreneur to undertake the higher effort if the threat of non-renewal of the financing in case of failure is sufficiently probable, i.e.:*

$$1 - \underline{p} > 1 - \overline{\overline{\underline{p}}} \tag{25}$$

Where $\overline{\overline{x}}(\delta)$ *and* $\overline{\overline{\underline{p}}}$ *are given in the appendix in (A39) and (A43) and verify the following relationships:*

$$\frac{\partial \overline{\overline{x}}(\delta, \underline{p})}{\partial \delta} > 0 \; ; \; \frac{\partial \overline{\overline{x}}(\delta, \underline{p})}{\partial \underline{p}} < 0 \tag{26}$$

$$\frac{\partial \overline{\overline{\underline{p}}}}{\partial (c_h - c_l)} < 0 \; ; \; \frac{\partial \overline{\overline{\underline{p}}}}{\partial \delta} > 0 \tag{27}$$

Proof. See the appendix.

From expression (26) it is clear that the likely is the threat of non-renewal of the financing (lower $\underline{p}$) the larger is the region of financial access. This means that there is a trade-off between increasing the region of financial access and ensuring the renewal of the financial relationship during the second period. However, expressions (27) reveal that this trade-off could be mitigated by making the threat of non-renewal less stringent for example by lowering the additional cost of higher effort or trying to increase the entrepreneur's foresightedness.

5. Comparison between the debt and sharing contracts

5.1. Economic inefficiency

I showed in proposition 4 that endowing the debt contract with the threat of non-renewal of the financing in case of failure is a necessary incentivizing mechanism. This generates an economic inefficient because even projects with positive net present value will be liquidated in case of failure at the end of the first period. In other words, there will be liquidation of a proportion of projects that failed because of the realization of the bad state of the nature, although the entrepreneur undertook the high level of effort in the first period and is willing to undertake similarly the high level of effort in the second period. In order to compare the level of this economic inefficiency let's consider that the economy comprises a continuum of entrepreneurs of mass 1 situated uniformly along the interval [0,1]. Each entrepreneur $i \in [0,1]$ is initially endowed with an amount of capital equals to $f^i = (1 - x_0^i)F$. In order to simplify the presentation let's assume that x_0^i "equals" the index of the entrepreneur i which is equivalent to say that the entrepreneurs are ordered on $[0,1]$ increasingly with their needs of external financing. In order to determine the economic inefficiency of debt relatively to the sharing contracts let's consider the region where the two types of contracts are feasible without furthers conditions on the foresightedness of the entrepreneurs i.e.

$$x_0^i \in \left[0, \max(x_h, x_l^{'})\right] \cap \left[0, \max(\underline{x}, \hat{x})\right] \text{ or equivalently } x_0^i \in \left[0, x_{\min}\right]$$

where $x_{\min} = \min\{\max(x_h, x_l^{'}), \max(\underline{x}, \hat{x})\}$. In the presence of the above mentioned incentive, the entrepreneur undertakes the higher effort during the first period to increase the probability of success of its project which equals θ_h. Since the risks of the projects are identical and independent, according to the law of large numbers the proportion of successful projects in the region $\left[0, x_{\min}\right]$ is θ_h and the failing projects represent a proportion of $1 - \theta_h$. Besides, let's recall that the financier offering a debt financial contract renews the financing of the successful projects with certainty and but refinances the failing projects only with probability $\underline{p}$. Therefore, among the proportion $1 - \theta_h$ of the failing projects only a proportion $\underline{p}$ will be refinanced. Hence, the total investment during the second period in this case is $I_2^d = \left(\theta_h + \underline{p}\left(1 - \theta_h\right)\right) x_{\min} F$. Whereas, total investment under the sharing contract is given by $I_2^{sh} = x_{\min} F$ since all the projects even those having failed during the first period will be refinanced. Therefore, the economic inefficiency of the debt contract could be defined as the percentage of decrease in the investment relatively to the alternative sharing financing relationship:

$$\ell^{inv} = \left| \frac{I_2^d - I_2^{sh}}{I_2^{sh}} \right| \times 100 = (1 - \theta_h)(1 - \underline{p}) \times 100 \tag{28}$$

Expression (28) shows clearly that the economic inefficiency of the debt contract is increasing with the stringency of the threat of non-renewal of the financial relationship which is captured by $(1 - \underline{p})$. For example, if the probability of success is $\theta_h = 0.8$ and the probability of refinancing in case of failure is $\underline{p} = 0.3$ then the economic inefficiency of debt relatively to the sharing contract equals 14%. If we calculate the economic inefficiency of debt contract in term of the second-period output loss we obtain the following expression:

$$\ell^{out} = \frac{E_\pi^h}{F} \ell^{inv} \geq (1 + \rho) \ell^{inv} \tag{29}$$

Where the equality in (29) is derived from the relationship between investment and expected payoff[6] and the inequality obtained using (4). Therefore, for a risk-free rate of return equal to 10% the economic inefficiency in term of output loss becomes 15.4%.

5.2. Access to finance

I showed in proposition 1 and 2 that both the sharing and debt contract are feasible when the internal funds (provided by the entrepreneur) are superior to determined thresholds (see also figure 3). These thresholds depend on the characteristics of the project (size, payoffs, and probability of success/failure) and the opportunity cost of the financier. In this section, I generate numerical simulations to compare the two contracts in terms of financial access.

5.2.1. Effect of the variation of the probability of success

I consider a project of size F =200 which generate a high payoff $\overline{\pi} = 1.98F$ in case of success and low payoff $\underline{\pi} = 0.22F$ in case of failure. The probability of success in case of low effort is $\theta_l = 0.46$ and I vary the probability of success in case of high effort as follows $\theta_h \in [0.55;1]$. I consider that the risk-free rate of return is $\rho = 5\%$ and the discount factor of the entrepreneur is $\delta = 0.95$. Moreover, the cost of low effort is $c_l = 0.1(E_\pi^h - E_\pi^l)$ whereas the cost of high effort is $c_h = 0.4(E_\pi^h - E_\pi^l)$ which ensures that condition (3) is satisfied. In addition to the variation of the probability of success θ_h in case of high effort I also vary the proportion x_0 of external financing needs in the range $[0.1\%,100\%]$. The following table summarizes the different values of the parameters while figures 6.a, 6.b, 7.a and 7.b illustrate the results of the simulations.

$\rho = 5\%$	$F = 200$	$\underline{\pi} = 0.22F$	$c_l = 0.1(E_\pi^h - E_\pi^l)$	$\theta_h \in [0.55;1]$
$\delta = 0.95$	$\theta_l = 0.46$	$\overline{\pi} = 1.98F$	$c_h = 0.4(E_\pi^h - E_\pi^l)$	$x_0 \in [0.1\%,100\%]$

[6] The expected payoff (output) of one project in case of higher effort undertaken by the entrepreneur and which is obtained from an investment F is given by equation (2) : $E_\pi^h = \theta_h \overline{\pi} + (1-\theta_h)\underline{\pi} > F$. Therefore, the output in case of a proportion η of independent projects is given by $y = \left(\eta\theta_h\overline{\pi} + \eta(1-\theta_h)\underline{\pi}\right) = \eta E_\pi^h$ while the total investment is $I = \eta F$. Therefore, the relationship between output and investment is given by $y = \left(\eta\theta_h\overline{\pi} + \eta(1-\theta_h)\underline{\pi}\right) = \frac{E_\pi^h}{F} I$.

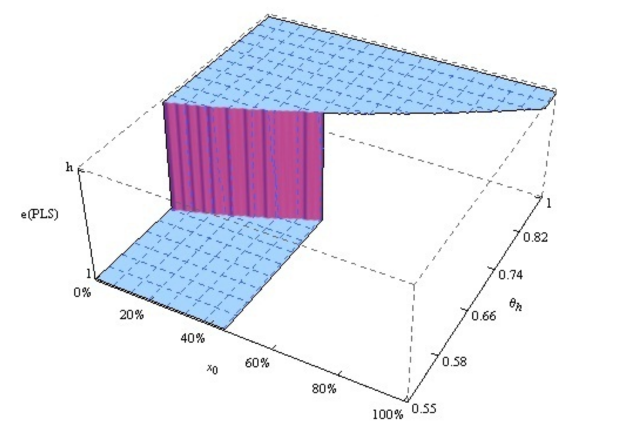

Figure 6.a. Effort, financial access and the probability of success in case of profit sharing contract and one-period relationship.

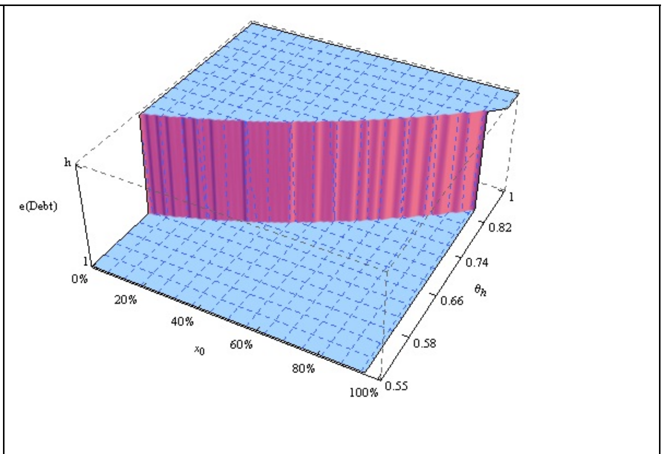

Figure 6.b. Effort, financial access and the probability of success in case of debt contract and one-period relationship.

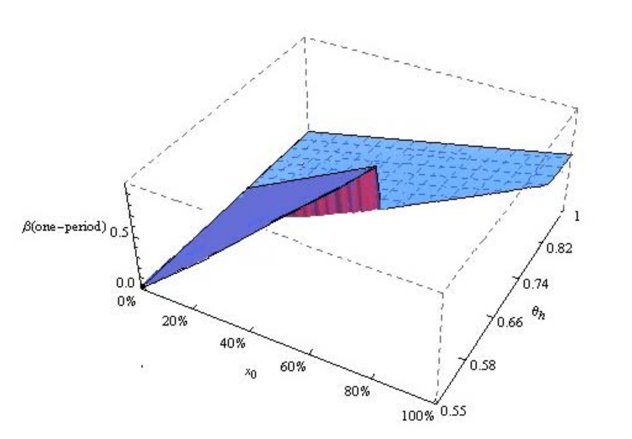

Figure 7.a. Effect of the external financing and the probability of success on the share ratio of the financier in case of profit sharing contract and one-period relationship.

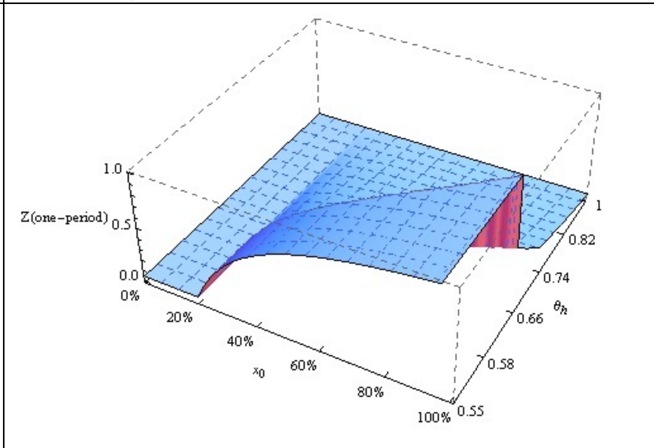

Figure 7.b. Effect of the external financing and the probability of success on the interest rate (Z-1) in case of debt contract and one-period relationship.

Figure 6.a and 6.b show that the debt contract is characterized by larger financial access than the profit sharing contract. More precisely the profit sharing contract is not feasible when the external financial needs are superior to 45% of the cost of the project and the probability of success in case of higher effort is inferior to 0.8. Therefore, the debt contract is more suitable for project with relatively higher risk and low internal funding. The two figures show also that, for the two types of contract, the higher effort is undertaken by the entrepreneur only for sufficiently level of the higher probability of success. This threshold is itself increasing with the external financing need which means that higher external needs may induce the entrepreneur to choose the lower effort whereas for the same type of projects another entrepreneur with higher internal funds chooses the higher effort. Figure 7.a. and 7.b. present the profit sharing ratio β and the interest rate Z .

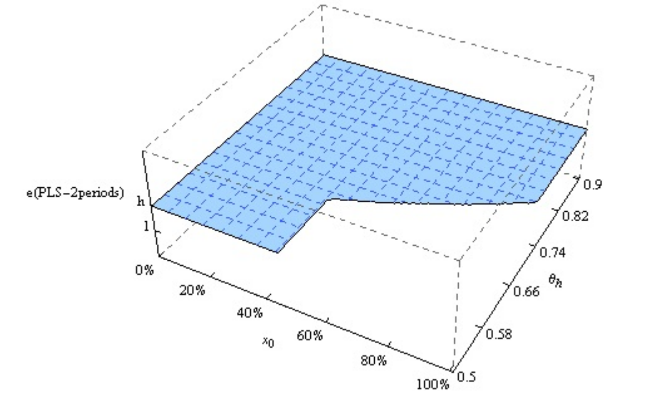

Figure 8.a. Effort, financial access and the probability of success in case of profit sharing contracts and two-period relationship.

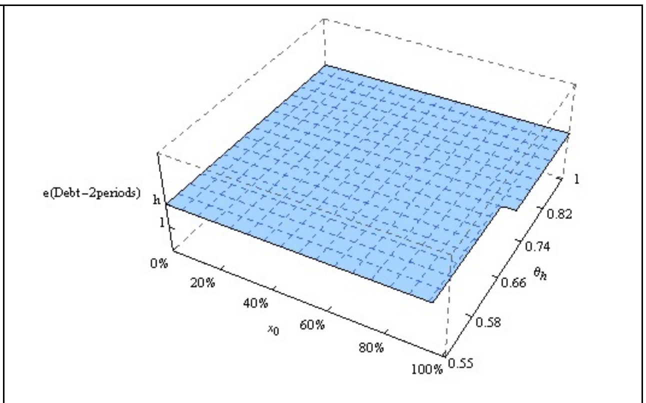

Figure 8.b. Effort, financial access and the probability of success in case of debt contracts and two-period relationship, and a probability $\underline{p} = 0$.

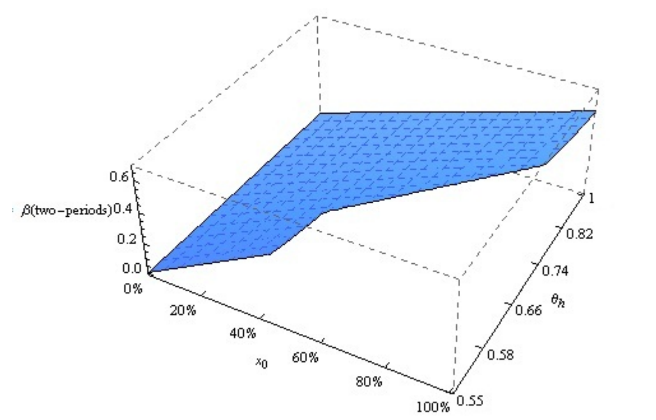

Figure 9.a. Effect of the external financing and the probability of success on the share ratio of the financier in case of profit sharing contract and two-period relationship.

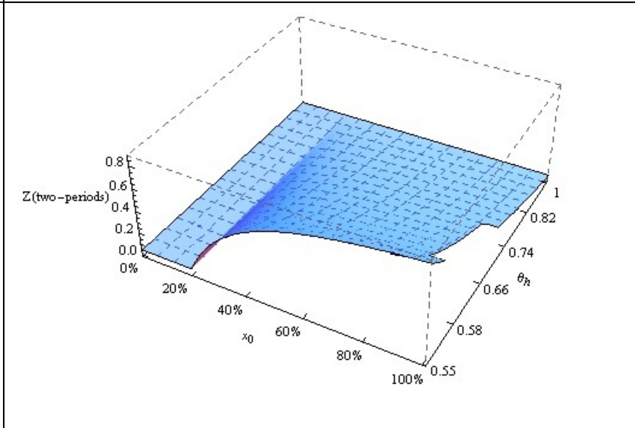

Figure 9.b. Effect of the external financing and the probability of success on the interest rate (Z-1) in case of debt contract and two-period relationship, and a probability $\underline{p} = 0$.

Figure 8.a and 8.b show that the relationship between the financier and the entrepreneur over two periods enlarges the region of feasibility of the two contracts and reduces the gap between the profit sharing and the debt contract although the latter continues to be characterized by wider financial access. However, let's note that the simulation was done with a probability $1 - \underline{p} = 1$ which signifies that an entrepreneur whose project failed during the first period will have no chance to access to external financing under the debt contracting relationship. According to Proposition 4 this corresponds to the larger possible region of financial access under the debt contract. Therefore, the gap in terms of financial access between the sharing and the debt contract will be lower for any other debt contract. It is also interesting to note that this longer relationship horizon induces the entrepreneur to choose the higher effort. Figure 9.a and 9.b show that the financing costs of the two financial contracts are reduced mainly due to the possibility of the contracts to incentivize the entrepreneur to undertake the higher effort.

5.2.2. Effect of the variation of the risk-free return

I consider a project of size F =3000 which generate a high payoff $\overline{\pi} = 2.09F$ in case of success and low payoff $\underline{\pi} = 0.001F$ in case of failure. The probability of success in case of low effort is $\theta_l = 0.45$ and the probability of success in case of high effort is $\theta_h = 0.8$. I vary the risk-free rate of return ρ in the interval $[1\%;30\%]$. The discount factor of the entrepreneur is $\delta = 0.95$. Moreover, the cost of low effort is $c_l = 0.1(E_\pi^h - E_\pi^l)$ whereas the cost of high effort is $c_h = 0.8(E_\pi^h - E_\pi^l)$ which ensures that condition (3) is satisfied. In addition to the variation of the risk-free rate of return ρ, I also vary the proportion x_0 of external financing needs in the range $[0.1\%,100\%]$. The following table summarizes the different values of the parameters while figures 10.a, 10.b, 11.a and 11.b illustrate the results of the simulations.

$F = 3000$	$\theta_l = 0.45$	$\underline{\pi} = 0.001F$	$c_l = 0.1(E_\pi^h - E_\pi^l)$	$\rho \in [1\%;30\%]$
$\delta = 0.95$	$\theta_h = 0.8$	$\overline{\pi} = 2.09F$	$c_h = 0.8(E_\pi^h - E_\pi^l)$	$x_0 \in [0.1\%,100\%]$

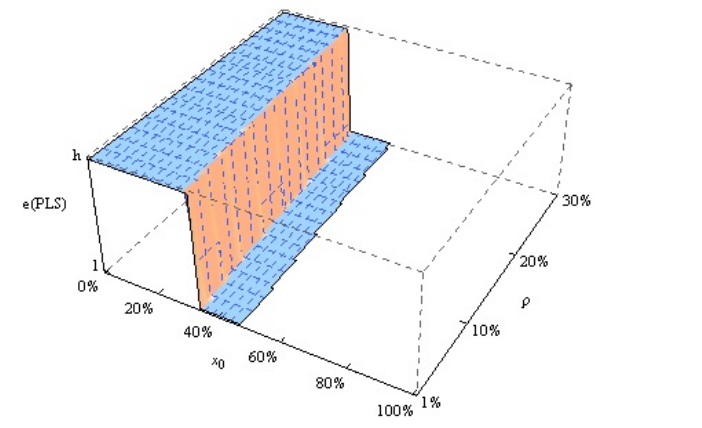

Figure 10.a. Effort, financial access and the risk-free return in case of profit sharing contract and one-period relationship.

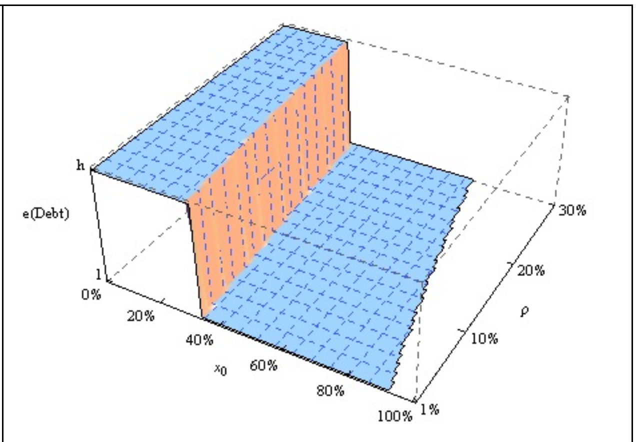

Figure 10.b. Effort, financial access and the risk-free return in case of debt contract and one-period relationship.

Again, figure 10.a and 10.b show that the debt contract is characterized by larger financial access than the profit sharing contract in case of one-period relationship. Indeed, the two contracts are not feasible when the external financial needs are superior to determined thresholds which are itself decreasing with the opportunity cost of the financier. However, the threshold of the profit sharing is inferior to that characterizing the debt contract.

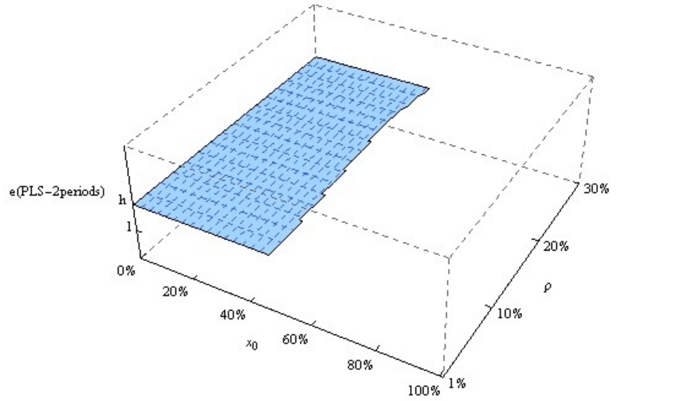

Figure 11.a. Effort, financial access and the risk-free return in case of profit sharing contracts and two-period relationship.

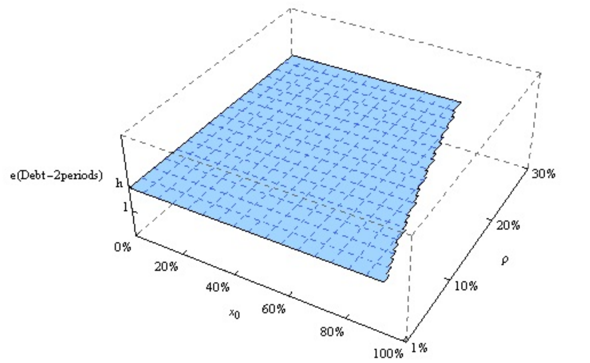

Figure 11.b. Effort, financial access and the risk-free return in case of debt contracts and two-period relationship with a probability $\underline{p} = 0$.

Figure 11.a and 11.b confirm the results of the simulations illustrated by figures 10.a and 10.b which are the positive effect of the longer horizon on the incentive of the entrepreneur to undertake higher effort and also on the region of financial access.

6. Policy implications

It is well known that one of the explanations of the low use by Islamic banks of the sharing modes of finance relatively to “mark-up” modes is the difficulty to deal with the agency problems (moral hazard and adverse selection) (Siddiqui, 2006). According to Ul Haque and Mirakhor (1987, p161) “bankers ascribe the problem of moral hazard or asymmetric information to be an important explanation for individual preference for short-term liquidity.” This problem is even deepen is countries suffering from weak legal systems and higher cost of contract enforcement. The results obtained in the context of one-period financial relationship (section 3) show that it is possible to extend the use of the sharing contract by widening the financial access to the entrepreneurs through two policies or a mix of them. The first policy consists in subsidizing the cost of higher effort. The higher effort should be interpreted broadly and could reflect the use of a modern technology by the entrepreneur which necessitates additional expenses and learning process. A special governmental fund subsidizing the adoption of the higher effort by the entrepreneurs endowed with low internal resources (SMEs) could be one of the tools to implement this policy. Figure 12 illustrates the effect of this policy which reduces the minimum internal resources required for the entrepreneur to access to external financing. These minimum resources decreases from $1 - \underline{x}$ to $1 - \underline{x}^s$. Figure 13 illustrates the effect of the second policy which consists in reducing the opportunity cost of the financier which equals the risk-free rate of return ρ . The reduction of the risk-free rate could be done through higher taxation of the financier’s revenues generated through “risk-free” financial operations relatively to the revenues generated through the “sharing” financial operations. A taxation advantaging the “sharing” contract could be justified in terms of reduction of the economic inefficiency which has been identified in the context of debt contract and two-periods. Figure 13 illustrates the effect of taxation on the additional benefit of higher effort. The tax's effect has been chosen (for explanation reason) to generate the same effect decreasing the minimum resources required for financial access from $1-\underline{x}$ to $1-\underline{x}^s$.

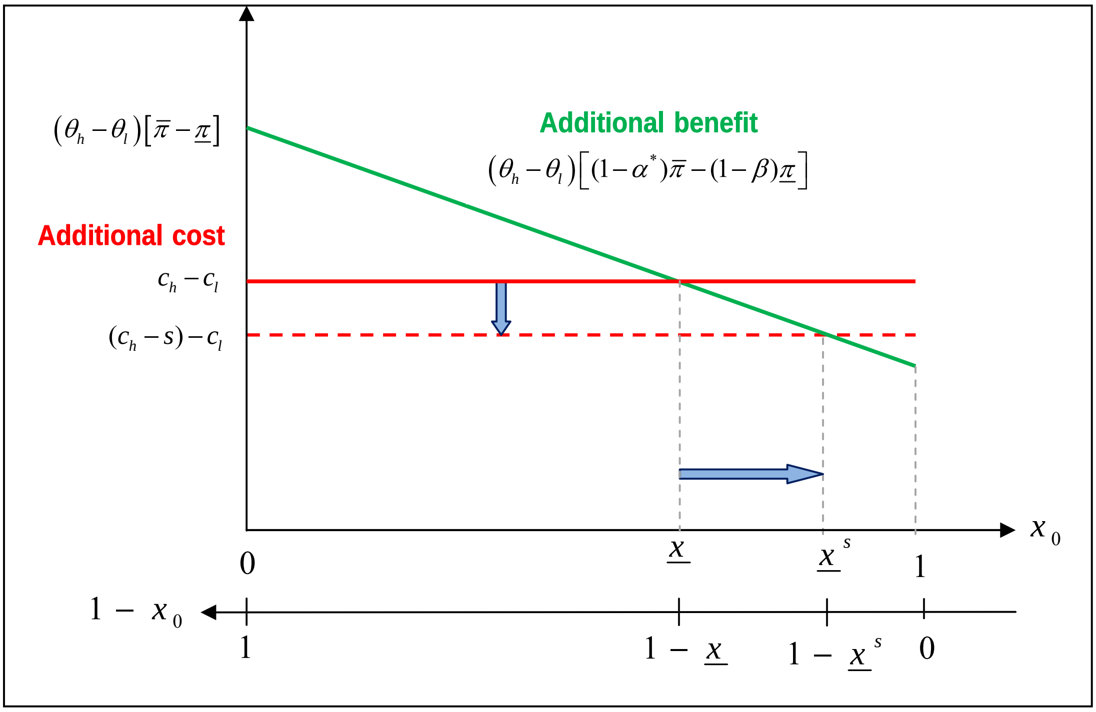

Figure 12. Effect of subsidizing the cost of higher effort

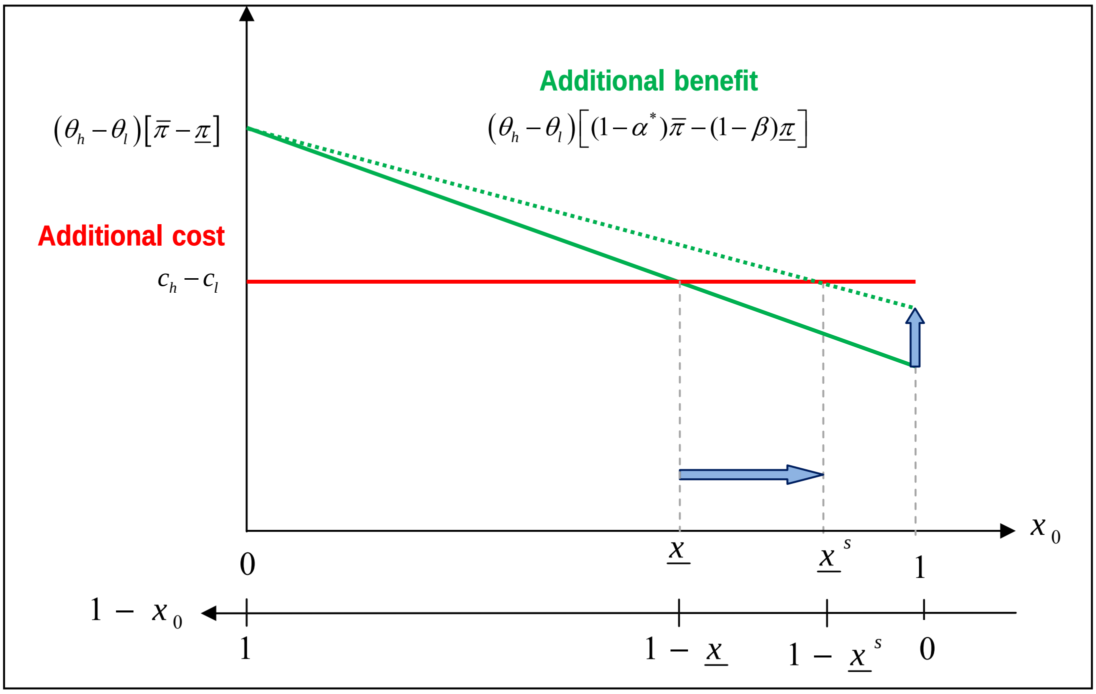

Figure 13. Effect of reducing the risk-free rate of return (through taxation for example)

A policy-mix consisting in a combination of taxation of the “risk-free” financial operations and subsidizing of the “higher effort” has the advantage to reduce the amplitude of the adjustment made through each policy. As illustrated in figure 14, a combination of lower subvention of the higher effort compared to figure 12 and lower taxation compared to figure 13 generates the same effect (relatively to individual policy) on the enlargement of the region of financial access.

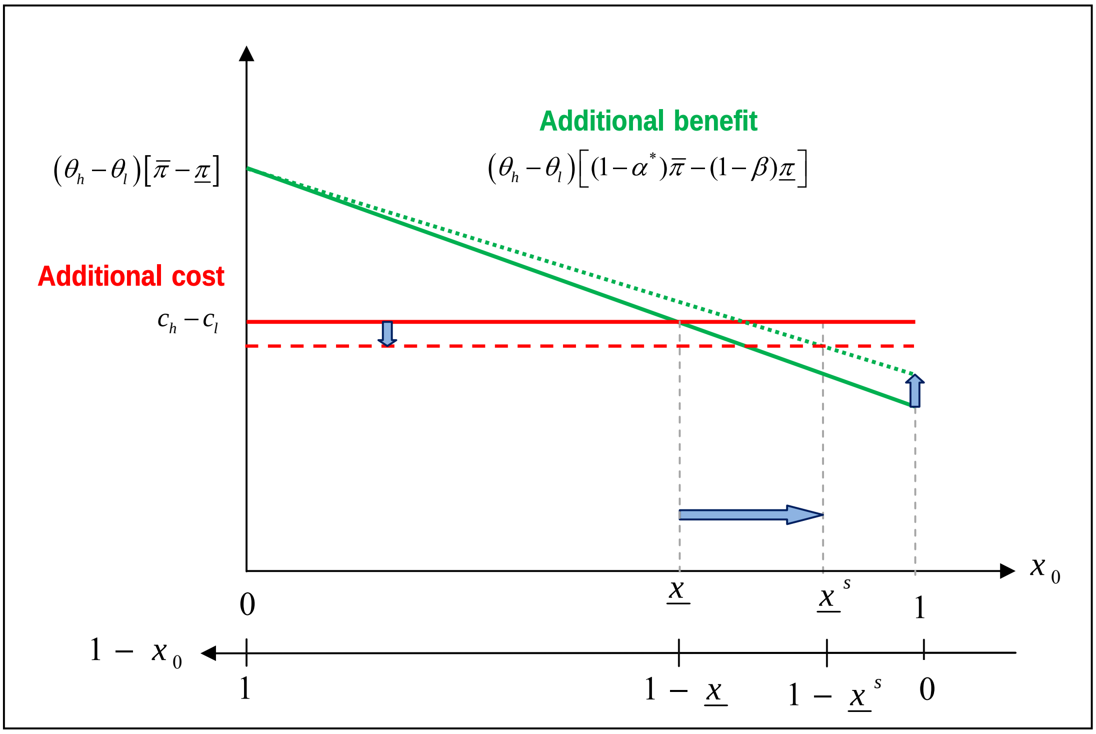

Figure 14. Effect of a mix-policy of subsidizing and taxation

The results obtained in the context of two-period financial relationship (section 4) show that it is possible to enlarge the region of financial access (through the sharing and debt contracts) through the longer horizon of financial relationship from one period to two periods. It has also been shown that for the larger horizon to have a positive impact, the entrepreneurs need to be sufficiently foresighted. In this context, a governmental agency that accompanies the entrepreneurs with low financial resources, by providing them with economic incentives on a larger horizon is expected to have a positive externality on the financier-entrepreneur relationship under the sharing and debt contracts.

7. Extension

In addition to their ex-post economic inefficiency debt contracts are characterized by the fact that all the outcome of the project is transferred to the financier in case of bad performance $(\underline{\pi})$ even if the entrepreneur has undertook the higher effort (see figure 4). While the debt contract is more beneficial than the profit sharing contract in case of high payoff of the project.

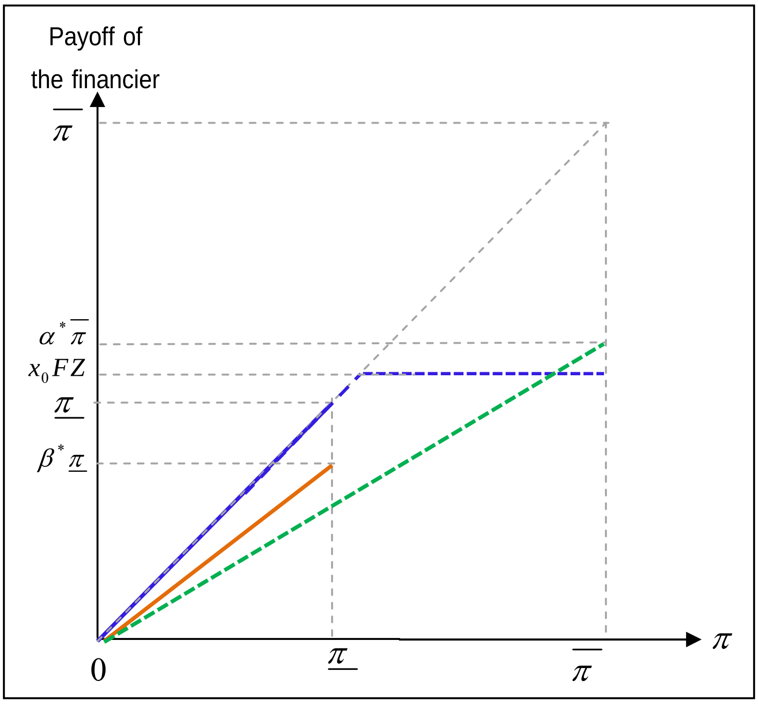

Figure 15. Comparison of the payoffs for the two types of contracts

It is clear that from the perspective of the risk neutral entrepreneur, the debt contract is ex-ante preferable to the profit sharing contract for two reasons. The first one is due to the fact that the risk neutral entrepreneur maximizes its expected wealth. The second reason is the fact that for the debt contract, earning of the high payoff is more likely than earning of the lower payoff (the probability θ_h is superior to the probability $1-\theta_h$). The situation is symmetric for the sharing contract. The ex-post analysis is different, since an entrepreneur who has undertaken the higher effort but whose project failed prefer to obtain a proportion of the payoff rather than having nothing to earn. The ex-ante analysis will be different in case of a risk-averse entrepreneur whose utility decreases with the volatility of the revenues. In this case, the tradeoff between increasing the expected revenue and reducing the volatility of the revenue will favor the sharing contract when the risk aversion exceeds a given threshold. An extension of the present model would be to analyze the Pareto optimality of the sharing contract relatively to the debt contract in the context of two-periods, a risk-neutral financier and risk-averse entrepreneur.

7. Conclusion

The objective of this paper is to compare sharing (equity) and debt contracts in presence of moral hazard which manifests as the hidden effort undertaken by the entrepreneur. The originality of this paper relatively to the existing studies consists in performing the comparison between the two types of contracts while considering a more general context along two dimensions. The first dimension is enabling the internal funds provided by the entrepreneur to vary between null (which corresponds to Innes, 1990 and the other above mentioned studies) and a level just inferior to 100%. The second dimension is the incorporation of an incentive mechanism to the sharing (equity) contract in the context of a two-period relationship. The incentive mechanism is related to the fact that the entrepreneur's (financier') share in the project's payoff is increasing (decreasing) with the internal funds.

I showed that the sharing and debt contracts are feasible when the internal funds of the entrepreneur are superior to determined thresholds. These thresholds depend on the characteristics of the project (size, payoffs, and probability of success/failure) and the opportunity cost of the financier. The debt contract is shown to be characterized by larger financial access than the sharing contract. I have also shown that the enlargement of the financial-relationship to two periods has an incentivizing effect on the entrepreneur and enlarges the region of financial access for the two types of contracts, if a common condition of sufficiently foresighted entrepreneur is satisfied. However, two distinct conditions are also necessary for the enlargement of the financial access to occur. For the sharing contract, the second condition is related to the size of the project which should be inferior to a determined threshold. For the debt contract, the second condition is related to the threat of non-renewal of the financing in case of first-period failure, which should be sufficiently stringent. In addition, it has been shown that the more restrictive the threat of non-renewal the larger the region of financial access. However, this is realized at the expense of the second period investment which decreases, and represents the economic efficiency' effect of the debt contract. In the policy recommendation section, I discussed the effect of taxing the "risk-free" financial operation and subsidizing the "higher effort" of the insufficiently-capitalized entrepreneurs on the enlargement of the financial access.

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Appendix

Proof of Lemma 1

The entrepreneur undertakes the lower level of effort after signing the contract if this increases his utility. This is the case if the following condition holds

$$\mathrm{E}(\tilde{\mathrm{W}}^{ent}) = \theta_l\left(1-\alpha^*\right)\overline{\pi} + (1\text{-}\theta_l)\left(1-\beta^*\right)\underline{\pi} - c_l > \mathrm{E}(\mathrm{W}^{ent}*) \tag{A1}$$

Using (7) condition (A1) becomes

$$(c_h - c_l) > \left(\theta_h - \theta_l\right)\left[(1-\alpha^*)\overline{\pi} - (1-\beta^*)\underline{\pi}\right] \tag{A2}$$

Injecting the expression (6) of α^* in (A2) and noting that $\beta^* = x$

$$\frac{(c_h - c_l)}{\left(\theta_h - \theta_l\right)} > \overline{\pi} - \underline{\pi} + \beta^*\underline{\pi} - \alpha^*\overline{\pi}$$

$$\frac{(c_h - c_l)}{\left(\theta_h - \theta_l\right)} - \left(\overline{\pi} - \underline{\pi}\right) > \beta^*\underline{\pi} - \alpha^*\overline{\pi} \tag{A3}$$

Let's show that $\alpha^*\overline{\pi} > \beta^*\underline{\pi}$. For that assume that $\alpha^*\overline{\pi} \le \beta^*\underline{\pi}$ and let's show that this is not possible. Multiplying the two members of this inequality by θ_h and adding $(1\text{-}\theta_h)\beta^*\underline{\pi}$ we obtain $\theta_h\alpha^*\overline{\pi} + (1-\theta_h)\beta^*\underline{\pi} \le \beta^*\underline{\pi}$. Recalling that equation (5) gives $\theta_h\alpha^*\overline{\pi} + (1-\theta_h)\beta^*\underline{\pi} = (1+\rho)xF$ we obtain that $(1+\rho)xF \le \beta^*\underline{\pi}$ which becomes $(1+\rho)F \le \underline{\pi}$ since $\beta^* = x$. This is clearly in contradiction with the assumption that $F > \underline{\pi}$. Therefore, we can now return to (A3) to obtain

$$\frac{\left(E_\pi^h - E_\pi^l\right) - (c_h - c_l)}{\left(\theta_h - \theta_l\right)} < \alpha^* \overline{\pi} - \beta^* \underline{\pi} = x\left[\overline{\pi} - \underline{\pi} - \frac{E_\pi^h - (1+\rho)F}{\theta_h}\right]$$ or equivalently

$$x > \underline{x} = \frac{(E_\pi^h - E_\pi^l) - (c_h - c_l)}{\left(1 - \theta_l / \theta_h\right)\left[\theta_h(\overline{\pi} - \underline{\pi}) + (1+\rho)F - E_\pi^h\right]} = \theta_h \frac{\overline{\pi} - \underline{\pi} - (c_h - c_l)/(\theta_h - \theta_l)}{(1+\rho)F - \underline{\pi}} = \theta_h \frac{(\theta_h - \theta_l)(\overline{\pi} - \underline{\pi}) - (c_h - c_l)}{\left((1+\rho)F - \underline{\pi}\right)(\theta_h - \theta_l)}$$

$$x > \underline{x} == \theta_h \frac{\left(E_\pi^h - E_\pi^l\right) - \left(c_h - c_l\right)}{\left((1+\rho)F - \underline{\pi}\right)(\theta_h - \theta_l)}$$

Proof of Proposition 1

i) From Lemma 1 we know that if $(1-x) \geq (1-\underline{x})$ then the entrepreneur will undertake the higher effort. In this case, the entrepreneur's expected rate of return is identical to the case of symmetric information and equals ρ.

ii-1) From Lemma 1 we know that if $(1-x) < (1-\underline{x})$ then the entrepreneur chooses the lower effort if he is offered the partnership contract $(e = h, xF, \alpha^*, \beta^*)$. The expected rate of return of the financier is determined by the following equation

$$\mathrm{E}(\tilde{\mathrm{W}}^{inv}) = \theta_l \alpha^* \overline{\pi} + (1\text{-}\theta_l)\beta^* \underline{\pi} = (1+\tilde{\rho})x\ F \tag{A4}$$

Which gives us using (6)

$$\tilde{\rho} = \rho - \left(\frac{\theta_h - \theta_l}{xF}\right)\left(\alpha^* \overline{\pi} - \beta^* \underline{\pi}\right) \tag{A5}$$

ii-2) For the expected rate of return of the financier to equal ρ although the entrepreneur chooses the lower effort we should have the following equation

$$\mathrm{E}(\hat{\mathrm{W}}^{inv}) = \theta_l \hat{\alpha} \overline{\pi} + (1\text{-}\theta_l)\beta^* \underline{\pi} = (1+\rho)x\ F \tag{A6}$$

Recalling equation (5):

$$\mathrm{E}(\mathrm{W}^{inv}*) = \theta_h \alpha^* \overline{\pi} + (1\text{-}\theta_h)\beta^* \underline{\pi} = (1+\rho)xF$$

It is easy to show that

$$\hat{\alpha} = \frac{\theta_h}{\theta_l}\alpha^* + \frac{\theta_l - \theta_h}{\theta_l}\beta^* \frac{\underline{\pi}}{\overline{\pi}} = \frac{\theta_h \overline{\pi}\alpha^* + \left(\theta_l - \theta_h\right)x\underline{\pi}}{\theta_l \overline{\pi}} \tag{A7}$$

From equation (6) we have $\alpha^* = x\left[1 - \frac{E_\pi^h - (1+\rho)F}{\theta_h \overline{\pi}}\right] = x\left[\frac{\theta_h \overline{\pi} - \left(\theta_h - \theta_l\right)\left(\overline{\pi} - \underline{\pi}\right) + (1+\rho)F}{\theta_h \overline{\pi}}\right]$

Injecting this expression of α^* in (A7) we found

$$\hat{\alpha} = x\frac{\theta_h\overline{\pi} - (\theta_h - \theta_l)(\overline{\pi} - \underline{\pi}) + (1+\rho)F + (\theta_l - \theta_h)\underline{\pi}}{\theta_l\overline{\pi}} \tag{A8}$$

$$= x\left(1 + \frac{(1+\rho)F}{\theta_l\overline{\pi}}\right) = \alpha^*\left(1 + \frac{(1+\rho)F}{\theta_l\overline{\pi}}\right) > \alpha^*$$

However, we should ensure that $\hat{\alpha} \leq 1$ which is the case only if $x \leq \hat{x} = \frac{\theta_l\overline{\pi}}{\theta_l\overline{\pi} + (1+\rho)F} < 1$. Since we are under the case $x > \underline{x}$ the possibility for the financier to offer this contract is conditioned by the following condition $x \in \left]\underline{x}, \max(\underline{x}, \hat{x})\right]$. From Lemma 1 we know that if $(1-x) < (1-\underline{x})$ then the entrepreneur chooses the lower effort if he is offered the partnership contract $(e = h, xF, \alpha^*, \beta^*)$ since we have the inequality (A2) and given that $\hat{\alpha} > \alpha^*$ the same inequality (A2) holds for $\hat{\alpha}$ and the entrepreneur chooses the lower effort.

Proof of Proposition 2

The expected utility of the entrepreneur for an effort level $e \in \{h, l\}$ is given by

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e) = \theta_e(\overline{\pi} - xFZ) - c_e \tag{A9}$$

Therefore, the entrepreneur chooses the higher level of effort if $\mathrm{E}(\mathrm{W}^{ent} \setminus h) \geq \mathrm{E}(\mathrm{W}^{ent} \setminus l)$ or equivalently if the following condition is realized

$$\theta_h(\overline{\pi} - xFZ) - c_h \geq \theta_l(\overline{\pi} - xFZ) - c_l$$

$$x \leq \tilde{x} = \frac{1}{FZ}\left(\overline{\pi} - \frac{c_h - c_l}{\theta_h - \theta_l}\right) \tag{A10}$$

Let's note that $\frac{\underline{\pi}}{FZ} < \tilde{x} < \frac{\overline{\pi}}{FZ}$. Hence, the financier's expected payoff is given by

$$\mathrm{E}(\mathrm{W}^{inv}) = \begin{cases} \mathrm{W}^{inv} = xFZ & \text{if } x \leq \frac{\underline{\pi}}{FZ} \\ \theta_h xFZ + (1-\theta_h)\underline{\pi} & \text{if } \frac{\underline{\pi}}{FZ} < x \leq \tilde{x} \\ \theta_l xFZ + (1-\theta_l)\underline{\pi} & \text{if } \tilde{x} < x \leq \frac{\overline{\pi}}{FZ} \end{cases}$$

The risk-neutral financier is willing to finance the firm if and only if the expected payoff equals $(1+\rho)xF$ the payment he obtains from a risk-free investment. This gives us the following condition on Z

$$\begin{cases} xFZ = (1+\rho)xF & \text{if } x \leq \frac{\underline{\pi}}{FZ} \\ \theta_h xFZ + (1-\theta_h)\underline{\pi} = (1+\rho)xF & \text{if } \frac{\underline{\pi}}{FZ} < x \leq \tilde{x} \\ \theta_l xFZ + (1-\theta_l)\underline{\pi} = (1+\rho)xF & \text{if } \tilde{x} < x \leq \frac{\overline{\pi}}{FZ} \end{cases}$$

$$\begin{cases} Z = Z_e = 1+\rho & \text{if } x \le \dfrac{\underline{\pi}}{FZ} \\ Z = Z_h = \dfrac{(1+\rho)xF - (1-\theta_h)\underline{\pi}}{\theta_h xF} & \text{if } \dfrac{\underline{\pi}}{FZ} < x \le \tilde{x} \\ Z = Z_l = \dfrac{(1+\rho)xF - (1-\theta_l)\underline{\pi}}{\theta_l xF} > Z_h & \text{if } \tilde{x} < x \le \dfrac{\overline{\pi}}{FZ} \end{cases}$$

Since the two last expressions of Z include x we should incorporate them in the bounds of the intervals in order to determine the latters. Using the expression of $\tilde{x}$ given by (A10) the inequalities $\underline{\pi} / FZ_h < x \le \tilde{x}$ are equivalent to

$$\frac{\underline{\pi}}{(1+\rho)F} < x \le x_h = \frac{E_\pi^h - \theta_h \dfrac{c_h - c_l}{\theta_h - \theta_l}}{(1+\rho)F} \tag{A11}$$

Similarly, the inequalities $\tilde{x} < x \le \overline{\pi} / FZ$ are equivalent to

$$\frac{E_\pi^l - \theta_l \dfrac{c_h - c_l}{\theta_h - \theta_l}}{(1+\rho)F} = x_l < x \le x_l^{'} = \frac{E_\pi^l}{(1+\rho)F} \tag{A12}$$

We have $x_h > x_l$ since $E_\pi^h - E_\pi^l > c_h - c_l$ but $x_h > x_l^{'}$ if and only if $\overline{\pi} > \underline{\pi} + \theta_h \dfrac{c_h - c_l}{\left(\theta_h - \theta_l\right)^2}$

Therefore, we have the following cases

- If $x \in \left[0, \dfrac{\underline{\pi}}{(1+\rho)F}\right]$ then the feasible contract is $(xF, (1+\rho)xF)$.
- If $x \in \left]\dfrac{\underline{\pi}}{(1+\rho)F}, x_l\right]$ then the feasible contract is $(xF, Z_h xF)$.
- If $x \in \left]x_l, \min(x_h, x_l^{'})\right]$ then there are two feasible contracts $(xF, Z_h xF)$ and $(xF, Z_l xF)$.
- If $x \in \left]\min(x_h, x_l^{'}), x_l^{'}\right]$ then the feasible contract is $(xF, Z_l xF)$.
- If $x \in \left]x_l^{'}, \max(x_h, x_l^{'})\right]$ then the feasible contract is $(xF, Z_h xF)$.
- $x \in \left]\max(x_h, x_l^{'}), 1\right]$ then there is no feasible contract.

A simple algebra shows that the entrepreneur prefers the contract $(xF, Z_h xF)$ with higher effort over the contract $(xF, Z_l xF)$ with lower effort when the two are feasible. Finally, the expressions of the parameters are summarized in the following:

$$x_l = \frac{E_\pi^l - \theta_l (c_h - c_l)/(\theta_h - \theta_l)}{(1+\rho)F},\ x_l^{'} = \frac{E_\pi^l}{(1+\rho)F},\ x_h = \frac{E_\pi^h - \theta_h (c_h - c_l)/(\theta_h - \theta_l)}{(1+\rho)F}$$

$$Z_h = \frac{(1+\rho)x_0F-(1-\theta_h)\underline{\pi}}{\theta_h x_0 F} > (1+\rho) \text{ and } Z_l = \frac{(1+\rho)x_0F-(1-\theta_l)\underline{\pi}}{\theta_l x_0 F} > Z_h \tag{A13}$$

Proof of Proposition 3

Before proving the results we need the following intermediary result which is easy to show using the equations (8) and (10).

Lemma2.

$$m\,\mathrm{ax}(\underline{x},\hat{x}) = \begin{cases} \underline{x} & \text{if } F < \underline{F} \\ \hat{x} & \text{if } F \geq \underline{F} \end{cases} \quad \textit{with}$$

$$\underline{F} = \left(\frac{\overline{\pi}\theta_l}{1+\rho}\right)\left[\left(\frac{\overline{\pi}\theta_l}{\theta_h\left(\overline{\pi}-\underline{\pi}-A\right)}\right)-1\right]^{-1} \quad \textit{and } A = \left(c_h - c_l\right)/(\theta_h-\theta_l) \tag{A14}$$

i) For the entrepreneur to undertake the higher effort during the first period,

$$\mathrm{E}(\mathrm{W}^{ent} \backslash e_1 = h, e_2) \geq \mathrm{E}(\mathrm{W}^{ent} \backslash e_1 = l, e_2) \tag{A15}$$

with

$$\mathrm{E}(\mathrm{W}^{ent} \backslash e_1 = h, e_2) = -c_h + \delta\left(\theta_{e_2} E\left((1-\hat{\alpha}_2)\backslash e_1 = h\right)\overline{\pi} + \left(1-\theta_{e_2}\right)E(1-x_1)\underline{\pi} - c_{e_2}\right) \tag{A16}$$

$$\mathrm{E}(\mathrm{W}^{ent} \backslash e_1 = l, e_2) = -c_l + \delta\left(\theta_{e_2} E\left((1-\hat{\alpha}_2)\backslash e_1 = l\right)\overline{\pi} + \left(1-\theta_{e_2}\right)E(1-x_1)\underline{\pi} - c_{e_2}\right) \tag{A17}$$

For the entrepreneur to choose the higher effort during the first period we have to obtain
Using (A16) and (A17) the condition (A15) becomes

$$c_h - c_l \leq \delta\theta_{e_2}\overline{\pi}\left[E(1-\hat{\alpha}_2 \backslash e_1 = h) - E(1-\hat{\alpha}_2 \backslash e_1 = l)\right] \tag{A18}$$

(A17) signifies that the additional effort cost should be more than compensated by the increase in the expected discounted wealth. From equation (22) we have

$$E(1-\hat{\alpha}_2 \backslash e_1 = h) - E(1-\hat{\alpha}_2 \backslash e_1 = l) = \left(\theta_h - \theta_l\right)\left((1+\rho)F - (1-\theta_h)\underline{\pi}\right)\frac{\left(1-\hat{\alpha}_1\right)\overline{\pi} - \left(1-x_0\right)\underline{\pi}}{\theta_h F}$$

Injecting the last equation in (A17) we obtain

$$\frac{c_h - c_l}{\theta_h - \theta_l} \leq \delta\theta_{e_2}\frac{\left(1-\hat{\alpha}_1\right)\overline{\pi} - \left(1-x_0\right)\underline{\pi}}{\theta_h F}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right]$$

$$\frac{c_h - c_l}{\theta_h - \theta_l} \leq \frac{\delta\theta_{e_2}}{\theta_h F}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right]\left[\left(1-\hat{\alpha}_1\right)\overline{\pi} - \left(1-x_0\right)\underline{\pi}\right]$$

$$\frac{c_h - c_l}{\theta_h - \theta_l} \leq \frac{\delta\theta_{e_2}}{\theta_h F}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right]\left[\overline{\pi} - \underline{\pi} + x_0\underline{\pi} - \hat{\alpha}_1\overline{\pi}\right]$$

According to (20) we have $\widehat{\alpha}_1\overline{\pi} = x_0\left(\dfrac{(1+\rho)F-(1-\theta_h)\underline{\pi}}{\theta_h}\right)$. Therefore (A16) is also equivalent to

$$\frac{c_h-c_l}{\theta_h-\theta_l} \le \frac{\delta\theta_{e_2}}{\theta_h F}\left[(1+\rho)F-(1-\theta_h)\underline{\pi}\right]\left[\overline{\pi}-\underline{\pi}-\frac{(1+\rho)F-\underline{\pi}}{\theta_h}x_0\right]$$

We consider that the incentive for the entrepreneur to undertake the higher effort during the second period holds ($e_2 = h$) therefore we obtain

$$\left(\frac{(1+\rho)F-\underline{\pi}}{\theta_h}\right)x_0 \le \overline{\pi}-\underline{\pi}-\frac{F}{\delta}\frac{\dfrac{c_h-c_l}{\theta_h-\theta_l}}{\left[(1+\rho)F-(1-\theta_h)\underline{\pi}\right]}$$

which becomes

$$x_0 \le \tilde{x} = \underline{\underline{x}} - \frac{\tilde{\delta}}{\delta}\left(\underline{\underline{x}}-\underline{x}\right) \tag{A19}$$

with $\underline{x}$ given by (8) and the rest of the variables are

$$\underline{\underline{x}} = \frac{\theta_h\left(\overline{\pi}-\underline{\pi}\right)}{(1+\rho)F-\underline{\pi}} > \underline{x} \tag{A20}$$

$$\tilde{\delta} = \frac{F}{\left[(1+\rho)F-(1-\theta_h)\underline{\pi}\right]} \tag{A21}$$

Therefore, the partnership contract over two periods enlarges the region of financial access if $\tilde{x} > m\mathrm{ax}(\underline{x},\hat{x})$. Using lemma 2 this is equivalent to the following conditions

$$\begin{cases}\tilde{x} > \underline{x} & \text{if } F < \underline{F} \\ \tilde{x} > \hat{x} & \text{if } F \ge \underline{F}\end{cases} \Leftrightarrow \begin{cases}\underline{\underline{x}} - \dfrac{\tilde{\delta}}{\delta}\left(\underline{\underline{x}}-\underline{x}\right) > \underline{x} & \text{if } F < \underline{F} \\ \underline{\underline{x}} - \dfrac{\tilde{\delta}}{\delta}\left(\underline{\underline{x}}-\underline{x}\right) > \hat{x} & \text{if } F \ge \underline{F}\end{cases} \Leftrightarrow \begin{cases}\delta > \tilde{\delta} & \text{if } \ F < \underline{F} \\ \delta\dfrac{\underline{\underline{x}}-\hat{x}}{\underline{\underline{x}}-\underline{x}} > \tilde{\delta} & \text{if } \ F \ge \underline{F}\end{cases} \tag{A22}$$

Using (A20) and (10) it is easy to show that $\underline{\underline{x}} - \hat{x} > 0$ is possible if and only if

$$F < \underline{\underline{F}} = \frac{\overline{\pi}\theta_h\theta_l}{1+\rho}\frac{\overline{\pi}-\left(1+1/\theta_h\right)\underline{\pi}}{\left(\theta_h-\theta_l\right)\overline{\pi}+\theta_h\underline{\pi}} \tag{A23}$$

Therefore (A22) becomes

$$\begin{cases}\delta > \tilde{\delta} & \text{if } \ F < \underline{F} \\ \delta > \dfrac{\underline{\underline{x}}-\underline{x}}{\underline{\underline{x}}-\hat{x}}\tilde{\delta} & \text{if } \ \underline{F} \ \le F \le \max(\underline{F},\underline{\underline{F}})\end{cases} \tag{A24}$$

ii) Now, for the entrepreneur to choose the higher effort during the second period we have to ensure that the following condition holds:

$$W_2^{ent}(e_1 = h, e_2 = h) \ge W_2^{ent}(e_1 = h, e_2 = l) \tag{A25}$$

$$\theta_h(1-\widehat{\alpha}_2)\overline{\pi}+(1-\theta_h)(1-x_1)\underline{\pi}-c_h\geq\theta_l\left(1-\widehat{\beta}_{h2}\right)\overline{\pi}+(1-\theta_l)(1-x_1)\underline{\pi}-c_l$$
$$(\theta_h-\theta_l)\left[(1-\widehat{\alpha}_2)\overline{\pi}-(1-x_1)\underline{\pi}\right]\geq c_h-c_l$$
$$(\theta_h-\theta_l)\left[\overline{\pi}-\underline{\pi}+x_1\underline{\pi}-\widehat{\alpha}_2\overline{\pi}\right]\geq c_h-c_l$$
Let's recall that equation (21) gives us
$$\widehat{\alpha}_2\overline{\pi}=x_1\left(\frac{(1+\rho)F-\underline{\pi}}{\theta_h}+\underline{\pi}\right)$$
Therefore (A24) becomes
$$\overline{\pi}-\underline{\pi}-x_1\left(\frac{(1+\rho)F-\underline{\pi}}{\theta_h}\right)\geq\frac{c_h-c_l}{\theta_h-\theta_l}$$
$$\overline{\pi}-\underline{\pi}-\frac{c_h-c_l}{\theta_h-\theta_l}\geq x_1\left(\frac{(1+\rho)F-\underline{\pi}}{\theta_h}\right)$$
Or equivalently $x_1\leq\overline{x}=(\theta_h-\theta_l)\underline{x}$ where x_1 is defined by (17) and (20) and given by

$$x_1=\begin{cases}1-\dfrac{x_0\underline{\pi}}{F}-\dfrac{x_0}{\theta_h}\left((1+\rho)-\dfrac{\underline{\pi}}{F}\right) & \text{with prob } \theta_h\\ 1-\dfrac{x_0\underline{\pi}}{F} & \text{with prob } 1-\theta_h\end{cases}$$

Therefore the condition for the entrepreneur to undertake the higher effort at the second period (whatever the state of the nature that occurs at the end of the first period) is the following
$\overline{x}\geq1-\dfrac{x_0\underline{\pi}}{F}$ or equivalently $x_0\geq\overline{x}'=\dfrac{F}{\underline{\pi}}\left(1-(\theta_h-\theta_l)\underline{x}\right)$

Let's now note that $\overline{x}'\leq1$ if and only if $F\geq\overline{F}=\dfrac{\theta_h\left(E_\pi^h-E_\pi^l-(c_h-c_l)\right)}{(1+\rho)(\theta_h-\theta_l)}+\dfrac{\underline{\pi}}{(1+\rho)}$

In other words if $F<\overline{F}$ then the entrepreneur undertakes the higher effort uncondionally. However, if $F\geq\overline{F}$ then the entrepreneur undertakes the higher effort only if $x_0\geq\overline{x}'$. Finally, the different thresholds presented in proposition 3 are summarized in the following:

$$\tilde{\delta}=\frac{F}{\left[(1+\rho)F-(1-\theta_h)\underline{\pi}\right]}\ ;\ \tilde{x}=\underline{\underline{x}}\left(1-\frac{\tilde{\delta}}{\delta}\right)+\frac{\tilde{\delta}}{\delta}\underline{x}\ ;\ \underline{\underline{x}}=\frac{\theta_h(\overline{\pi}-\underline{\pi})}{(1+\rho)F-\underline{\pi}}$$

$$\underline{F}=\left(\frac{\overline{\pi}\theta_l}{1+\rho}\right)\left[\left(\frac{\overline{\pi}\theta_l}{\theta_h\left(\overline{\pi}-\underline{\pi}-(c_h-c_l)/(\theta_h-\theta_l)\right)}\right)-1\right]^{-1} \tag{A26}$$

$$\underline{\underline{F}}=\frac{\overline{\pi}\theta_h\theta_l}{1+\rho}\frac{\overline{\pi}-(1+1/\theta_h)\underline{\pi}}{(\theta_h-\theta_l)\overline{\pi}+\theta_h\underline{\pi}}$$

Proof of Proposition 4

Before proving the results of this proposition we need the following intermediary result.

Lemma3.

$$\max(x_h, x_l^{'}) = \begin{cases} x_h & \text{if } c_h - c_l < \Delta c \\ x_l^{'} & \text{if } c_h - c_l \geq \Delta c \end{cases}$$

where $\Delta c = (\theta_h - \theta_l)^2 \left(\overline{\pi} - \underline{\pi}\right) / \theta_h$ (A27)

i) The expected wealth of the financier in each period should equal the wealth he would obtain if he invests his capital in a risk-free asset. At the end of the first period he obtains:

$$\widehat{\mathrm{W}}_1^{inv} = \min(x_0 FZ, \pi) = \begin{cases} x_0 FZ_1 & \text{with a probability of } \theta_{e_1} \\ \underline{\pi} & \text{with a probability of } 1\text{-}\theta_{e_1} \end{cases} \quad \text{(A28)}$$

Whereas at the end of the second period he obtains

$$\widehat{\mathrm{W}}_2^{inv} = \min(x_1 FZ_2, \pi) = \begin{cases} x_1 FZ_2 & \text{with a probability of } \theta_{e_2} \\ \underline{\pi} & \text{with a probability of } 1\text{-}\theta_{e_2} \end{cases} \quad \text{(A29)}$$

Therefore,

$$\mathrm{E}(\widehat{\mathrm{W}}_1^{inv} \setminus e_1 = h, l) = \theta_{e_1} x_0 FZ_1 + (1\text{-}\theta_{e_1})\underline{\pi} = (1+\rho) x_0 F \quad \text{(A30)}$$

$$\mathrm{E}(\widehat{\mathrm{W}}_2^{inv} \setminus e_2 = h, l) = \theta_{e_2} x_1 FZ_2 + (1\text{-}\theta_{e_2})\underline{\pi} = (1+\rho) x_1 F \quad \text{(A31)}$$

From (A30) and (A31) we can easily derive the following expressions:

$Z_1 = Z_{e_1}(x_0)$ and $Z_2 = Z_{e_2}(x_1)$ (A32)

where $Z_{e=h,l}(x)$ is given by (A13).

The expected utility of the entrepreneur over the two periods is given by

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e_1, e_2) = -c_{e_1} + \delta\left\{\left[\theta_{e_1} + \underline{p}\left(1-\theta_{e_1}\right)\right]\left[\theta_{e_2}\left(\overline{\pi} - x_1 FZ_2\right) - c_{e_2}\right]\right\} \quad \text{(A33)}$$

For the entrepreneur to undertake the higher effort during the first period we have to obtain

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = h, e_2) \geq \mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = l, e_2) \quad \text{(A34)}$$

Where the two terms of the inequality are obtained from (A33) for $e_1 = h$ and $e_1 = l$ respectively

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = h, e_2) = -c_h + \delta\left[\theta_h + \underline{p}\left(1-\theta_h\right)\right]\left[\theta_{e_2}\left(\overline{\pi} - x_1 FZ_2\right) - c_{e_2}\right] \quad \text{(A35)}$$

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = l, e_2) = -c_l + \delta\left[\theta_l + \underline{p}\left(1-\theta_l\right)\right]\left[\theta_{e_2}\left(\overline{\pi} - x_1 F Z_2\right) - c_{e_2}\right] \tag{A36}$$

Using (A35) and (A36) the condition (A34) becomes

$$c_h - c_l \leq \delta\left(\theta_h - \theta_l\right)\left(1-\underline{p}\right)\left(\theta_{e_2}\left(\overline{\pi} - x_1 F Z_2\right) - c_{e_2}\right) \tag{A37}$$

which signifies that in order for the entrepreneur to undertake the higher effort during the first period, the additional effort cost should be more than compensated by the increase in the expected discounted wealth.

$$\frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} + c_h \leq \theta_{e_2}\overline{\pi} - \theta_{e_2} x_1 F Z_2$$

Since we have from (A31) $\theta_{e_2} x_1 F Z_2 \ = (1+\rho)x_1 F - (1-\theta_{e_2})\underline{\pi}$ then the condition (A37) becomes

$$\frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} + c_h \leq \theta_{e_2}\overline{\pi} - (1+\rho)x_1 F + (1-\theta_{e_2})\underline{\pi} + (1+\rho)F - (1+\rho)F$$

$$\leq \theta_{e_2}\overline{\pi} + (1-\theta_{e_2})\underline{\pi} + (1+\rho)F\left[1-x_1\right] - (1+\rho)F$$

But we have also $(1-x_1)F = \overline{\pi} - x_0 F Z_1$. Therefore, we obtain

$$\frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} + c_h \leq \theta_{e_2}\overline{\pi} + (1-\theta_{e_2})\underline{\pi} + (1+\rho)\left(\overline{\pi} - x_0 F Z_1\right) - (1+\rho)F$$

$$\frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} + c_h \leq \theta_{e_2}\overline{\pi} + (1-\theta_{e_2})\underline{\pi} + \frac{(1+\rho)}{\theta_h}\left(\theta_h\overline{\pi} - \theta_h x_0 F Z_1\right) - (1+\rho)F \tag{A38}$$

We have also from (A30): $\theta_h x_0 F Z_1$ $=(1+\rho)x_0 F - (1-\theta_h)\underline{\pi}$ which injected in (A38) gives us

$$\frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} + c_h \leq \theta_{e_2}\overline{\pi} + (1-\theta_{e_2})\underline{\pi} + \frac{(1+\rho)}{\theta_h}\left[\theta_h\overline{\pi} - (1+\rho)x_0 F + (1-\theta_h)\underline{\pi}\right] - (1+\rho)F$$

$$\frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} + c_h \leq \left[\theta_{e_2}\overline{\pi} + (1-\theta_{e_2})\underline{\pi}\right] + \left[\theta_h\overline{\pi} + (1-\theta_h)\underline{\pi}\right]\left[\frac{(1+\rho)}{\theta_h}\right] - \frac{(1+\rho)^2}{\theta_h} x_0 F - (1+\rho)F$$

$$\frac{(1+\rho)^2}{\theta_h} x_0 F \leq \left[E_\pi^{e_2} + E_\pi^h \frac{(1+\rho)}{\theta_h}\right] - (1+\rho)F - \frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} - c_h$$

$$x_0 \leq \frac{1}{F(1+\rho)}\left\{\left[\frac{\theta_h}{1+\rho}E_\pi^{e_2} + E_\pi^h\right] - \theta_h F - \left[\frac{\theta_h}{(1+\rho)}\right]\left[\frac{c_h - c_l}{\delta\left(1-\underline{p}\right)\left(\theta_h - \theta_l\right)} + c_h\right]\right\}$$

$$x_0 \le x_h + \frac{\theta_h}{(1+\rho)^2 F}\left(\left((1+\rho) - \frac{1}{\delta(1-\underline{p})}\right)\frac{(c_h - c_l)}{(\theta_h - \theta_l)} + \left(E_\pi^{e_2} - c_h - (1+\rho)F\right)\right) \tag{A39}$$

$$x_0 \le \overline{\overline{x}}(\delta, \underline{p}) = x_h + \frac{\theta_h}{(1+\rho)^2 F}\left(\frac{c_h - c_l}{\theta_h - \theta_l}\right)\left(\frac{1}{\overline{\overline{\delta}}} - \frac{1}{\delta(1-\underline{p})}\right) \text{ where}$$

$$\overline{\overline{\delta}} = \frac{1}{\frac{\theta_h - \theta_l}{c_h - c_l}\left(E_\pi^{e_2} - c_h - (1+\rho)F\right) + (1+\rho)} \tag{A40}$$

Therefore, the debt contract over two periods enlarges the region of financial access if $\overline{\overline{x}}(\delta) > \max(x_h, x_l^{'})$. Using lemma 3 it is easy to show that this is equivalent to the following conditions

$$\begin{cases} \overline{\overline{x}}(\delta, \underline{p}) > x_h & \text{if } c_h - c_l < \Delta c \\ \overline{\overline{x}}(\delta, \underline{p}) > x_l^{'} & \text{if } c_h - c_l \ge \Delta c \end{cases} \Leftrightarrow \tag{A41}$$

$$\underline{p} < \overline{\overline{\underline{p}}} \quad \text{with} \tag{A42}$$

$$\overline{\overline{\underline{p}}} = \begin{cases} 1 - \frac{\overline{\overline{\delta}}}{\delta} & \text{if } c_h - c_l < \Delta c \\ 1 - \frac{\overline{\overline{\delta}}}{\delta}\frac{1}{1 - \overline{\overline{\delta}}(1+\rho)\left(1 - \Delta c/(c_h - c_l)\right)} & \text{if } c_h - c_l \ge \Delta c \end{cases} \tag{A43}$$

From (A39) it is easy to note that

$$\frac{\partial \overline{\overline{x}}(\delta, \underline{p})}{\partial \delta} > 0$$
$$\frac{\partial \overline{\overline{x}}(\delta, \underline{p})}{\partial \underline{p}} < 0$$

From (A43) we have also

$$\frac{\partial \overline{\overline{p}}}{\partial (c_h - c_l)} < 0$$

$$\frac{\partial \overline{\overline{p}}}{\partial \delta} > 0$$

ii) Now, for the entrepreneur to choose the higher effort during the second period we have to ensure that the following condition holds:

$$W_2^{ent}(e_1 = h, e_2 = h) \geq W_2^{ent}(e_1 = h, e_2 = l) \tag{A44}$$

Where the two terms of the inequality can be obtained from (A35) by using $e_2 = h$ and $e_2 = l$ respectively. By doing this (A44) becomes

$$\theta_h\left(\overline{\pi} - x_1 F Z_2\right) - c_h \geq \theta_l\left(\overline{\pi} - x_1 F Z_2\right) - c_l$$

$$c_h - c_l \leq \left(\theta_h - \theta_l\right)\left[\overline{\pi} - x_1 F Z_2\right] \tag{A45}$$

Let's note that

$$\delta\left(\theta_h - \theta_l\right)\left(1 - \underline{p}\right)\left(\theta_{e_2}\left(\overline{\pi} - x_1 F Z_2\right) - c_{e_2}\right) < \left(\theta_h - \theta_l\right)\left[\overline{\pi} - x_1 F Z_2\right]$$

Therefore, if condition (A37) holds the condition (A45) will be satisfied consequently. In other words, the condition that ensures that the entrepreneur undertakes the high effort during the first period ensures also that he has no incentive to shirk during the second period. Consequently, we can replace e_2 by h in (A41).